

Problem Sheet 5

Due April 4, 2001

Read Chapter 8 - 8.1-8.5 Battin
Chapter 2, 2.3 Vallado

1. The general three body problem gives stationary solutions in the plane of the rotating orbit which are along a straight line or in an equilateral triangle. The location of the points along a straight line can be found from Lagrange's quintic equation. The solution locates m_3 beyond m_2 assuming the distance between m_1 and m_2 is unity. The solution χ is the distance beyond m_2 (as observed from m_1). Lagrange's quintic equation is:

$$(m_1 + m_2) \chi^5 + (3m_1 + 2m_2) \chi^4 + (3m_1 + m_2) \chi^3 - (m_2 + 3m_3) \chi^2 - (2m_2 + 3m_3) \chi - (m_2 + m_3) = 0$$

For the case where $m_1 = 10$, $m_2 = 2$, and $m_3 = 1$,
Find the three straight line Lagrange points assuming the distance between m_1 and $m_2 = 1$ DU.
Note that all points can be found by clever use of the quintic equation.!

2. a) For the case where m_3 is negligible, and $m_1 + m_2 = 1$, the distance between them is 1 DU, (the restricted problem) and the ratio of $m_2/(m_1+m_2) = \mu$, determine the form of the quintic equation so that its coefficients are in terms of μ . (Note that this result will give the location of m_3 beyond m_2). Solve your derived equation for the location of m_3 beyond m_2 and compare it with the "exact solution found in problem 1.

b) Find a similar equation in terms of μ for the location of the point outside m_1 , determine its solution and compare it with that found in problem 1.

Note that it is possible to find a quintic equation for the restricted problem for each Lagrange point. The two outside points are easy, the one in the middle requires some heavy duty algebra.

3. The moon has a mass ratio with respect to the Earth of 1/81.3. The distance of the moon from the Earth is 384,400km. Assume its period is 27.32 days.

a) Determine the location of the three in-line Lagrange points in km for the Earth for the two adjacent to the Earth, and from the moon for the one on the other side of the moon.

b) Determine the period of the out of plane motion at each Lagrange point. (L1, ... L5)

c) Determine the periods of the in plane stable motions at each Lagrange point (including those at the "unstable" points).