AOE 5234 Orbital Mechanics, Problem Sheet 6

Due 16 April, 2001

1. Given the differential equation

$$\ddot{x} + x = R(t)$$

For R(t) = 0. The solution is $x = c_1 \sin t + c_2 \cos t$.

Using the variation of parameters, find:

a) The condition of osculation

b) The equation analogous to $\sum_{i=1}^{2} \frac{\partial \dot{\bar{r}}}{\partial \alpha_i} \dot{\alpha}_i = R$

c) The lagrangian brackets for this problem

d) Evaluate the 4 brackets

e) Find the differential equations for $\dot{c}_1$ and $\dot{c}_2$

f) Find the complete solution for arbitrary initial conditions x_0 and $\dot{x}_0$ for the cases:

i) $R(t) = A_0 \sin 5t$

ii) $R(t) = A_0 \sin t$

2. The equation

$$\dot{x} + x = \epsilon x^2$$

can be solved exactly using the variation of parameters. Assuming ϵ is small, it can also be solved approximately using the variation of parameters approach coupled with a straight forward perturbation technique.

a) Solve the above equation exactly using variation of parameters and integrating exactly he differential equation for the parameter.

b) Solve the above equation approximately to first order in ϵ using variation of parameters and a straight forward perturbation scheme to solve the differential equation for the parameter.

c) Show that if ϵ is small, the exact solution can be expanded in powers of ϵ and upon doing so the result will look the same as the result in part (b)

d) For a value of $\epsilon = 0.1$, compare the exact results with the approximate results for the case where the initial conditions are: at t = 0, x ,= 3.0. At what time will the error exceed 5% of the true value.?