C11. Incidence distribution for unswept tapered wings with an elliptic spanload.

This curiosity arose when I was making a study of the twist distribution required to obtain an elliptic spanload distribution for wings of midchord sweeps of 0, 15 and 30 degrees. I used John Lamar's design version of the old Rich Margason VLM code widely distributed by NASA Langley in the early 1970s (J. E. Lamar, NASA TN D-8090, June, 1976).

While I was running the code I thought of the formula from lifting line theory that can be derived for the twist distribution for unswept wings.

- How good is the lifting line theory formula? I've been using the formula since the mid 70s (40 years?). I haven't seen it written down in the form I use, but nobody has ever complained about it. It seems to be correct.
- Note also that I checked the sample case in Lamar's TN with the output from my version of the code, LamDes, (also evolved significantly since the mid 1970s). The result agreed with the values in the TN within a couple of percent. Also, John presented considerable verification of his code in the TN.

Review: Twist distribution for an elliptic load from lifting line theory.

We can find the twist distribution required for an elliptic spanload using lifting line theory. The monoplane equation provides a way to obtain this distribution, and is widely available in introductory aerodynamics texts, *i.e.*, Bertin and Cummings.

The monoplane equation is:

$$\mu(\alpha - \alpha_{OL})\sin\phi = \sum_{n=1}^{\infty} A_n \sin n\phi(\mu n + \sin\phi)$$

where, $\mu = \frac{c a_e}{8s}$, $s = \frac{b}{2}$, $c = c_{rt}[1-(1-\lambda)\eta]$, and a_e is the two-dimensional lift curve slope, assumed here to be 2π. Recall that only the first term in the series is non-zero for an elliptic spanload ($A_n = 0$ for n > 1), and $A_1 = \frac{C_L}{\pi AR}$.

We can relate y and ϕ to substitute into the relations above as follows:

$$y = -s\cos\phi \text{ or } y/s = -\cos\phi \text{ and } y/s = \eta, \text{so } \eta = -\cos\phi, \text{thus,}$$
$$\eta^2 = \cos^2\phi = 1 - \sin^2\phi, \textit{and } \sin^2\phi = 1 - \eta^2 \Rightarrow \sin\phi = \sqrt{1-\eta^2}$$

With a little algebra we get the final result:

- $$(\alpha - \alpha_{OL}) = \frac{C_L}{\pi AR}\left\{1 + \frac{(1+\lambda)AR\sqrt{1-\eta^2}}{\pi[1-(1-\lambda)\eta]}\right\}$$

Lifting line theory needs the value of alpha zero lift to find the value of alpha across the span. We will compare two cases. We use the classic NACA 6-series mean line chordloads and their related values of alpha zero lift. To compare with LamDes we choose two cases, one with a constant chordload, *a* = 1, and one with a triangular chordload, *a* = 0. The key values we need are available from Abbott and von Doenhoff, but I also checked them with DesCam and they agree. What is actually presented is the angle of attach for a lift coefficient of one. Assuming that the lift curve slope is 2π, we find that for the *a* = 0 chordload the α_{zl} is -9.11° and for the *a* = 1 chordload the α_{zl} is -4.55°.

The figure shows the comparison between lifting line theory and the VLM inverse method LamDes for these two cases. The agreement seems reasonable.