W. H. Mason, June 27, 2015

Curiosity Number 6. Adventures in Thin Airfoil Theory

I am reading the book by Doug McLean, *Understanding Aerodynamics*. He makes lots of interesting points and provides lots of food for thought. Although not really part of the big picture concepts he addresses, one figure caught my attention (of course I look at the pictures first before diving in). His figure 7.4.3b compares thin airfoil theory with a full solution from MSES. The poor agreement surprised me and I decided to take a look for myself (my mantra to students: be curious, be skeptical). His figure is given below as Fig. 1.

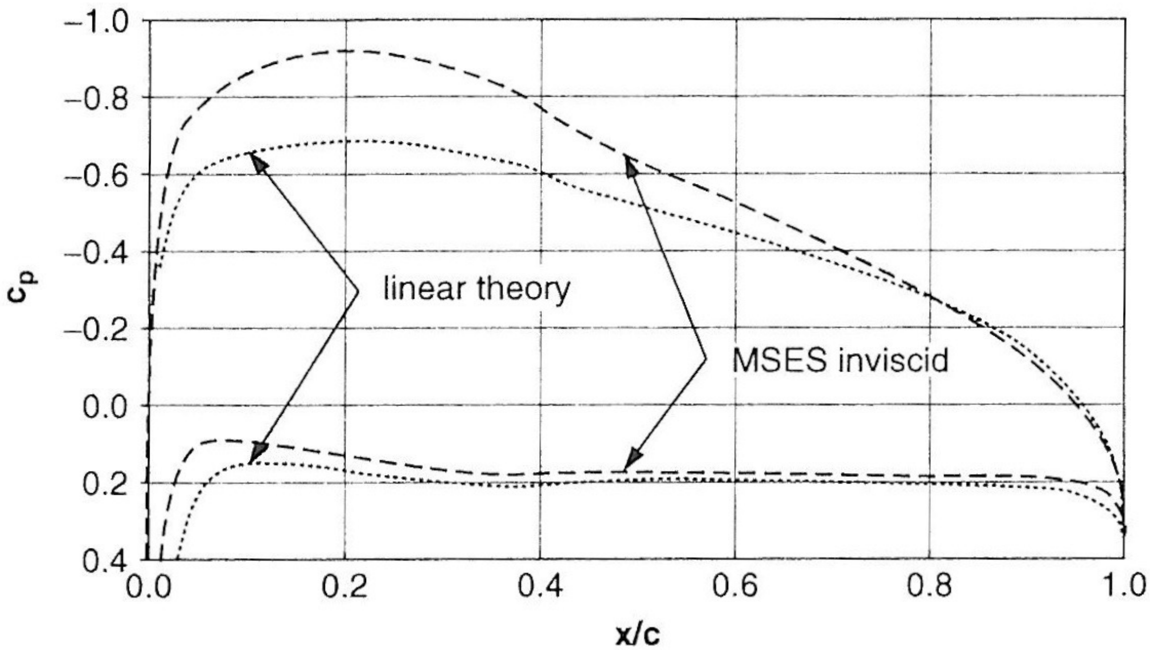

Figure 1. From McLean. Comparison of the pressure distributions between thin airfoil theory and a complete flowfield solution for an NACA 4410 airfoil at 2° alpha. (Of all the airfoils to pick, McLean picked one that apparently wasn't tested by the NACA. Why? Why not use the NACA 4412, one of the most widely used for comparison?)

In particular, I was surprised because of a figure that I recalled in Van Dyke, NACA R 1274, "Second Order Subsonic Airfoil Theory including Edge Effects." I thought Van Dyke showed pretty good agreement with an exact solution for an NACA 0012 airfoil at zero alpha (thickness comparison). As expected, the thin airfoil theory fails at the leading edge. A lot of Van Dyke's effort in matched asymptotic expansions eventually focused on how to deal with this. In Van Dyke's report the problem is treated using Riegels' Rule (we often used a version of this in the solution for wings when transonic small disturbance theory was the flow model). His result is shown in Fig. 2. Sure enough, for thickness the comparison between thin airfoil theory and the exact result looks pretty good except at the leading edge. This is much better than shown in Figure 1.

This led me to launch an investigation. Could McLean's result be right? (It turns out it is). This meant I had to compute it for myself, which led to a new understanding for me.

An observation: you hardly ever find pressure distribution results for thin airfoil theory. The key results since the time of Glauert have been the alpha for zero lift and the related pitching moment. I'd compared these predictions with data in the past and found really good agreement for zero lift alpha and of course the classic thin airfoil theory lift curve slope. How can these be so good when the pressure distribution looks so poor in Fig. 1?

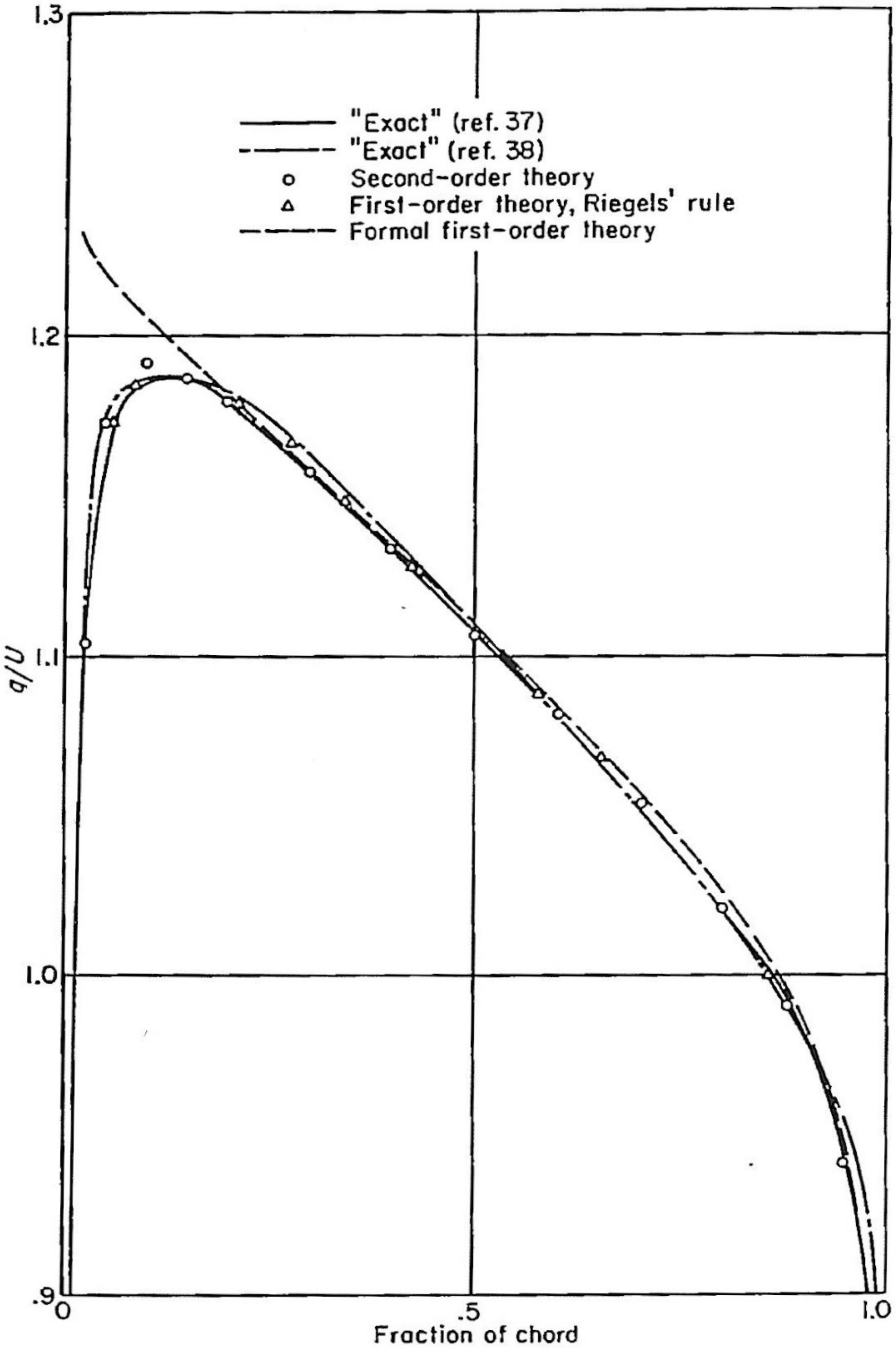

Figure 2. From Van Dyke (NACA R 1274). Comparison of surface speed predictions for an NACA 0012 airfoil at zero angle of attack.

Let's investigate. Using thin airfoil theory we check the thickness, camber and angle of attack contributions separately and then combine them. We'll do the thickness first, then review the lift curve work I've done before, and then look at the camber at zero alpha and finally the flat plate angle of attack. I had previously done parts of this, but I had to recreate some of the capabilities. At each step we will compare the predictions with other calculations (We will mainly be using panel methods and conformal transformation, thus you could ask the question – why bother? The answer is that if you're retired you can take the time to do this. I might learn something.)

Thickness

We can use the analytic solution given by Van Dyke for the NACA 00XX airfoils for thin airfoil theory. To check his solutions we run both a panel method and a conformal transformation solution. The results are given in Fig. 3. Since the airfoil is 10% thick instead of 12% thick the agreement should be better than the results shown in Fig. 2 (which were for surface velocity, not pressure).

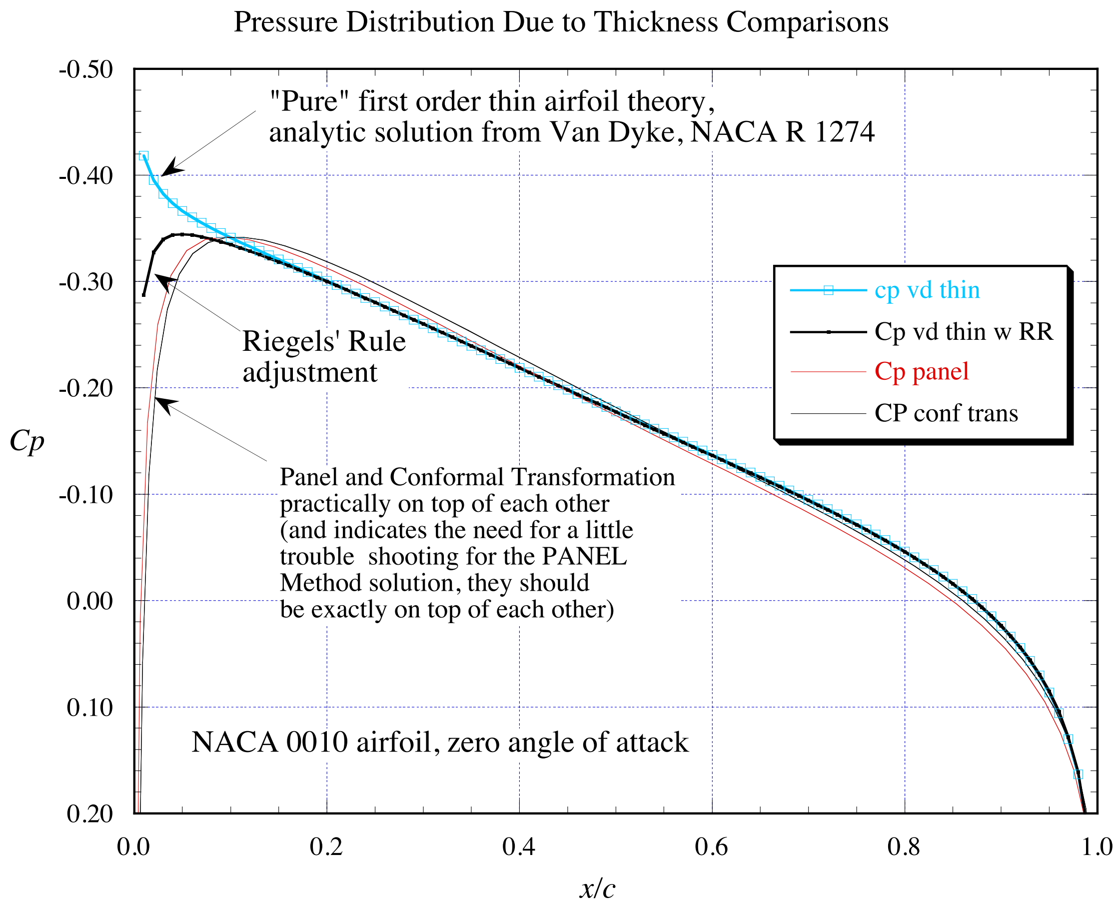

Figure 3. Comparison of pressures due to the basic thickness of the airfoil. Thin Airfoil theory compared to "exact" PANEL and Conformal Transformation methods. NACA 0010 airfoil at zero alpha.

The thin airfoil theory is pretty good, but underestimates the pressures slightly from about 10% to 50% of the chord. Since the ΔCp looks way off in Fig. 1, lets look at the lift curve comparison. It should be way off too. The lift must be way different between thin airfoil theory.

Next we look at the lift results from thin airfoil theory. The lift curve slope is 2π and the α for zero lift depends on the camber shape and magnitude. For NACA 4-digit cambers the analytical solution was worked out in Houghton and Carpenter[1] (I am looking at the 4th edition, Section 4.8.2). Figure 4 compares the thin airfoil theory result with wind tunnel tests (although this is from NACA R 669, 1939, the table is reproduced in Hemke, *Elementary Applied Aerodynamics*, where it's much more legible compared to the pdf file NASA put up on the web). I find this agreement to be remarkably good.

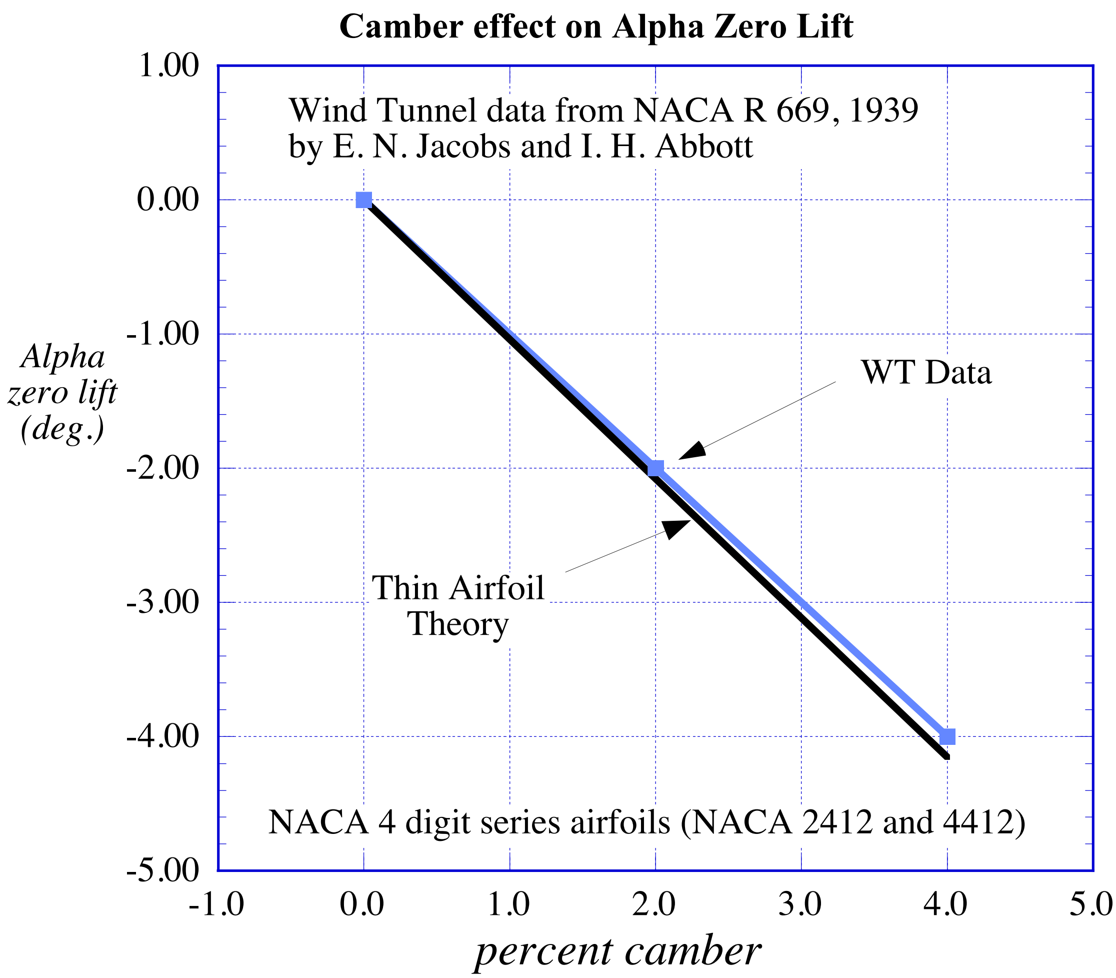

Figure 4. Comparison of the thin airfoil theory predicted zero lift angle of attack with wind tunnel tests for the NACA 4-digit camber lines at various values of the maximum camber.

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[1] The NACA 4412 is given as an example in the book. However, using the same (somewhat lengthy) formulas I get slightly different values for the results.

Now lets look at the resulting lift curves for thin airfoil theory compared to "exact" methods. We'll look at a 12% thick airfoil because I already have the results, including wind tunnel data (not available for the NACA 0010 airfoil). I realize that for McLean the comparison with experimental data is beside the point. Figure 5 shows the predictions from both a panel code (inviscid) and thin airfoil theory, as well as wind tunnel data. Curiously, the thin airfoil theory agrees with the test data better than the "exact" inviscid result (although completely fortuitous, I always get a kick out of students, and a couple of faculty at USAFA, running in to show me this result).

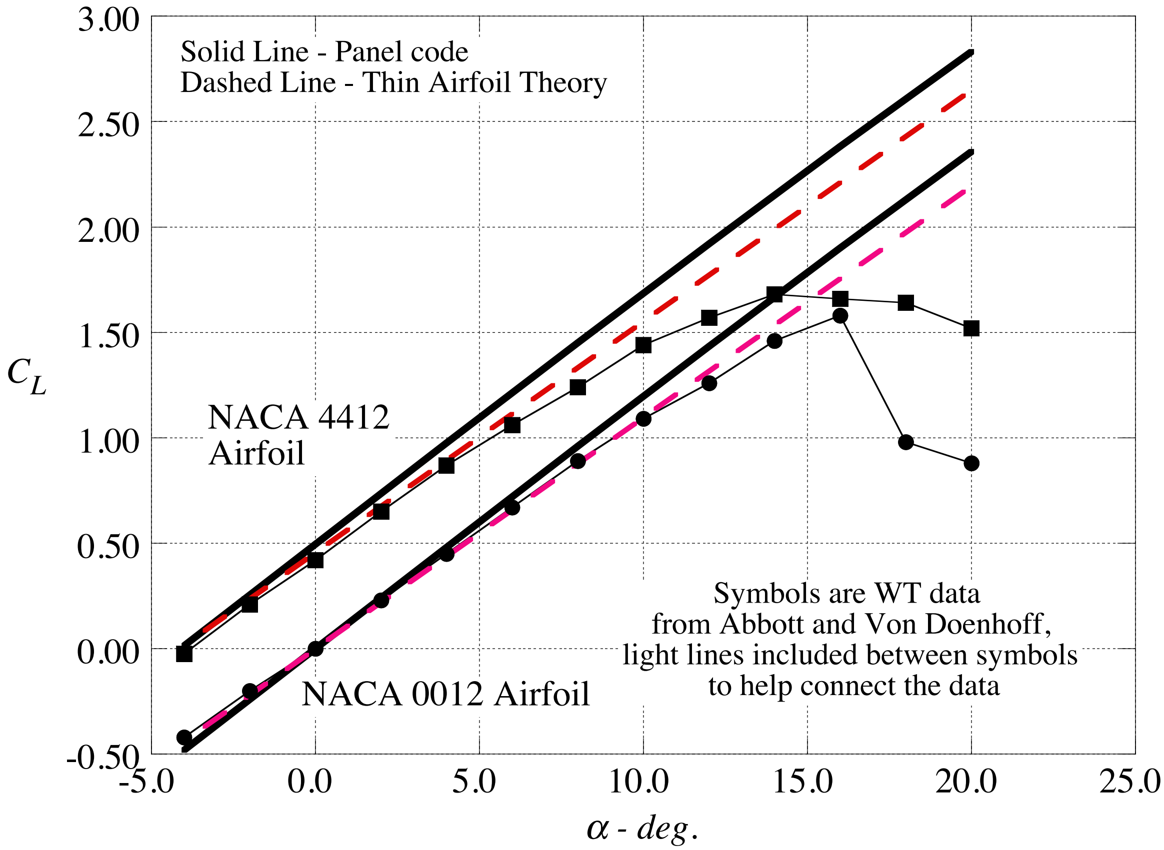

Figure 5. Comparison of the thin airfoil and exact panel method results for cambered and uncambered NACA 4-digit airfoils. Wind tunnel results are also included.

The lift at the value of two degrees alpha for the NACA 4412 airfoil looks closer to my eye than I would guess looking at the pressure distributions in Figure 1 (why I started this adventure). At two degrees alpha, the McLean case, the thin airfoil theory estimate for lift is 0.675. The NACA 4412 lift coefficient prediction from the Panel method is 0.7422, and the conformal transformation prediction is 0.7684. The corresponding values for the NACA 4410 are 0.7264 (Panel) and 0.7542 (conformal trans.). Thus the 12% thick airfoil lift coefficient inviscid prediction is slightly higher than for the 10% thick airfoil at 2° alpha (about 2%). For the 4410 case the difference between thin airfoil theory and the panel method is 7%.

Look at Fig. 6 to see how close the pressure coefficients are when there is a 3.7% lift coefficient difference between the panel and conformal transformation calculations. That's why I was surprised at the differences shown in Fig. 1.

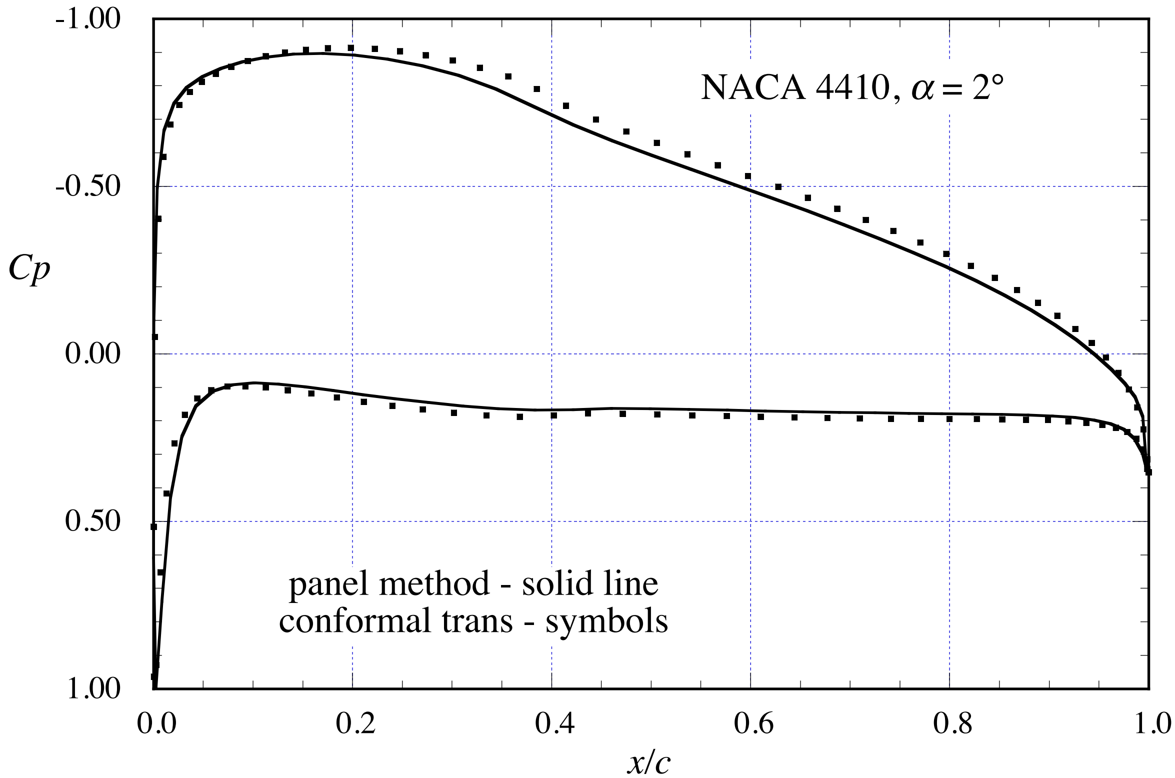

Figure 6. Panel and conformal transformation method results for the NACA 4410 airfoil at 2° alpha. The differences in the pressures appears pretty small.

Finally we'll look at my calculation of McLean's figure (Fig. 1). The result is given in Fig. 7. It looks pretty much exactly like McLean's figure, although I didn't modify the thin airfoil theory with a Riegels' Rule adjustment.

For the results shown in Fig. 7 the conformal transformation method C_L is 0.7542, while the thin airfoil theory prediction is 0.675 from the classical force solution, and from the integration of the so-called lumped vortex solution method[2] used to find the pressure distribution in Fig. 7 is 0.67492, in almost exact agreement with the analytical solution. The difference in lift coefficients is about 10.5%.

The somewhat surprising difference in pressures (to my eye) is apparently correct. And a 10% difference in lift is reasonable for first order thin airfoil theory compared to the "exact" conformal transformation.

[2] This is the name used by Katz and Plotkin for placing a point vortex at the ¼ chord of a panel and satisfying the boundary condition at the ¾ chord location on a panel.

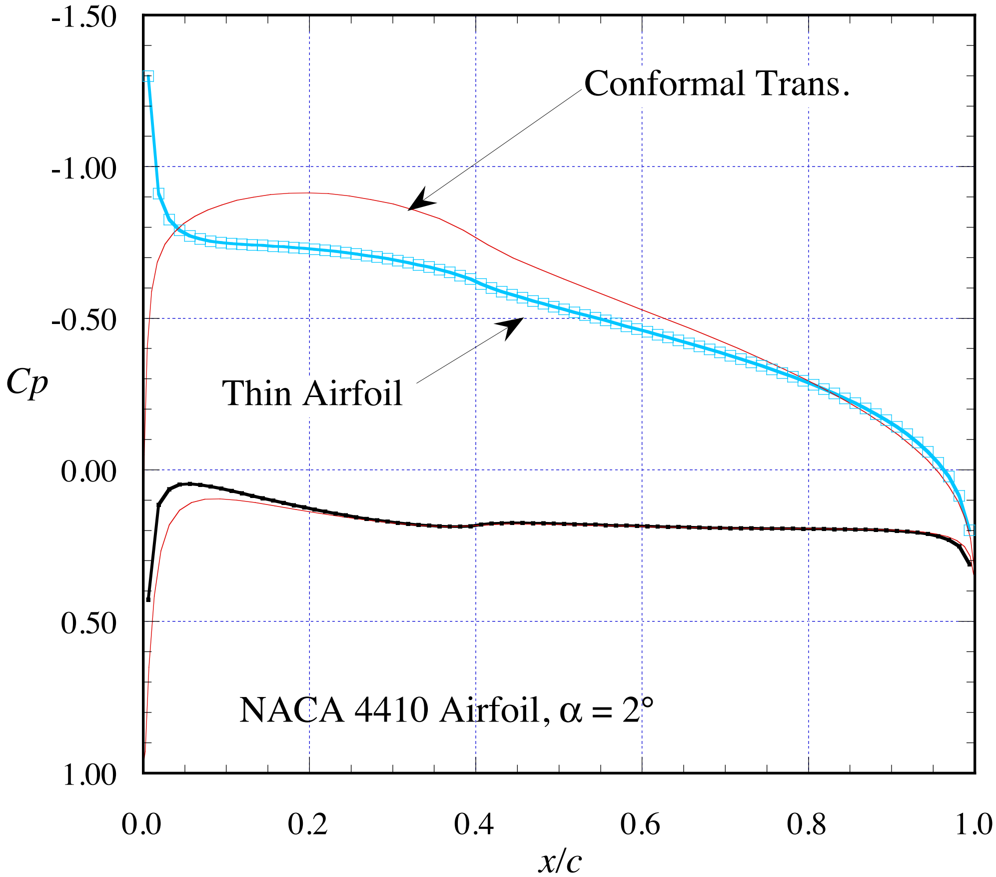

Figure 7. Comparison of the conformal transformation method and thin airfoil theory predictions for the pressure distribution on an NACA 4410 airfoil at $\alpha = 2°$.

It seems we need to look a little further into the comparison between thin airfoil theory and "exact" methods. Thus we compare the pressure distributions for a 10% thick uncambered NACA airfoil (NACA 0010). This is to see if there is an interaction between camber and thickness (suggested by Weber's second order theory[3]). Figure 8 shows this comparison. It was made for an angle of attack of 6 degrees, which produces a C_L roughly the same as the lift coefficient in Fig. 7. Although the trend of pressure distributions on the forward portion of the upper surface are similar to those seen above, to my eye the agreement is much better for this case than the cambered case. Now the lift coefficients differ by about 7% compared to the previous difference of about 10.5%. It looks to me like the thickness and flat plate angle of attack contributions are at least slightly more accurate than the camber contribution.

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[3] Weber (ARC R&M 3026) doesn't describe her method explicitly as second order, but that's how it's frequently described. There is a good table in Houghton and Boswell, *Further Aerodynamics for Engineering Students* that shows the components of her method (Fig. 2.20, pg 88).

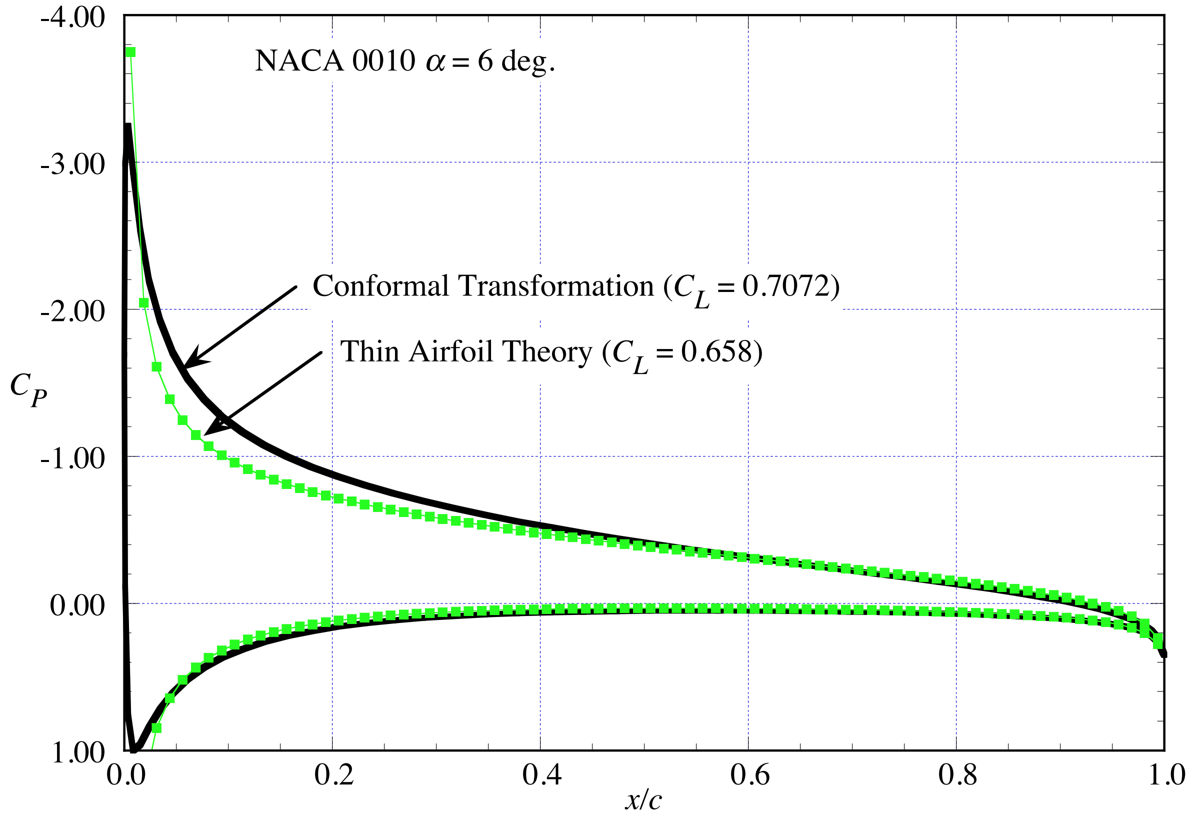

Fig. 8. Comparison of the pressure distributions for thin airfoil theory with conformal transformation results for an uncambered NACA 0010 airfoil at 6 deg. angle of attack.

Because the results of thin airfoil theory and conformal transformation in Fig. 8 appear to be in better agreement than shown in Fig. 1 or Fig. 7, we need to double check the camber calculation. To do this we try to validate the effect of camber on the solution in thin airfoil theory. The first chart is a check of my numerical calculation method with the analytic solution for a biconvex camber line since there is an analytic solution in just about every aerodynamics book. This is given in Fig 9. The results are acceptable for validation of the method, although they don't overlie perfectly. There is some room for interpretation converting the point vortex – control point solution method to ΔC_p. Note that we have previously shown (and it is also shown in Katz and Plotkin) that the difference between placing the point vortex on the actual camber line and satisfying the boundary condition on the actual camber line and placing the vortex on the axis and satisfying the boundary conditions on the axis is very small.

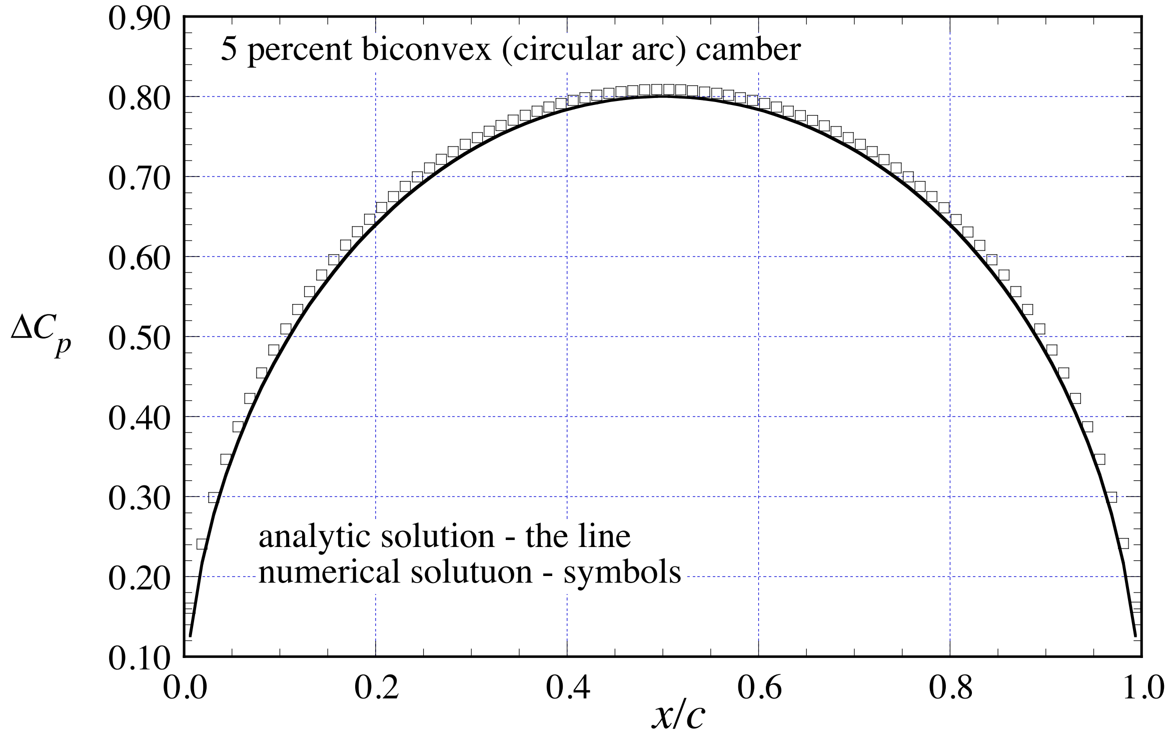

Figure 9. Comparison of analytic and numeric values of ΔC_p for a 5% max camber biconvex camber line (also shown in Katz and Plotkin)

Next, it is worth looking at the camber lines for the NACA 4-digit airfoils. In particular, we look at the slopes for various airfoil designations. The camber lines are made up of forward and aft parabolic arcs. We will look at the 64, 65 and 66 mean lines because these are the ones given in Abbott and vom Doenhoff. The first digit denotes the max camber. Other values are scaled from this value. The second digit specifies the location of the maximum value of the camber line. The slope variation is continuous, but with the exception of the NACA 65 series camber line (that corresponds to a biconvex camber line) there is a kink where they meet. (the change of slope at the intersection implies a jump in the curvature). Visually we see the camber line slopes in Fig. 10.

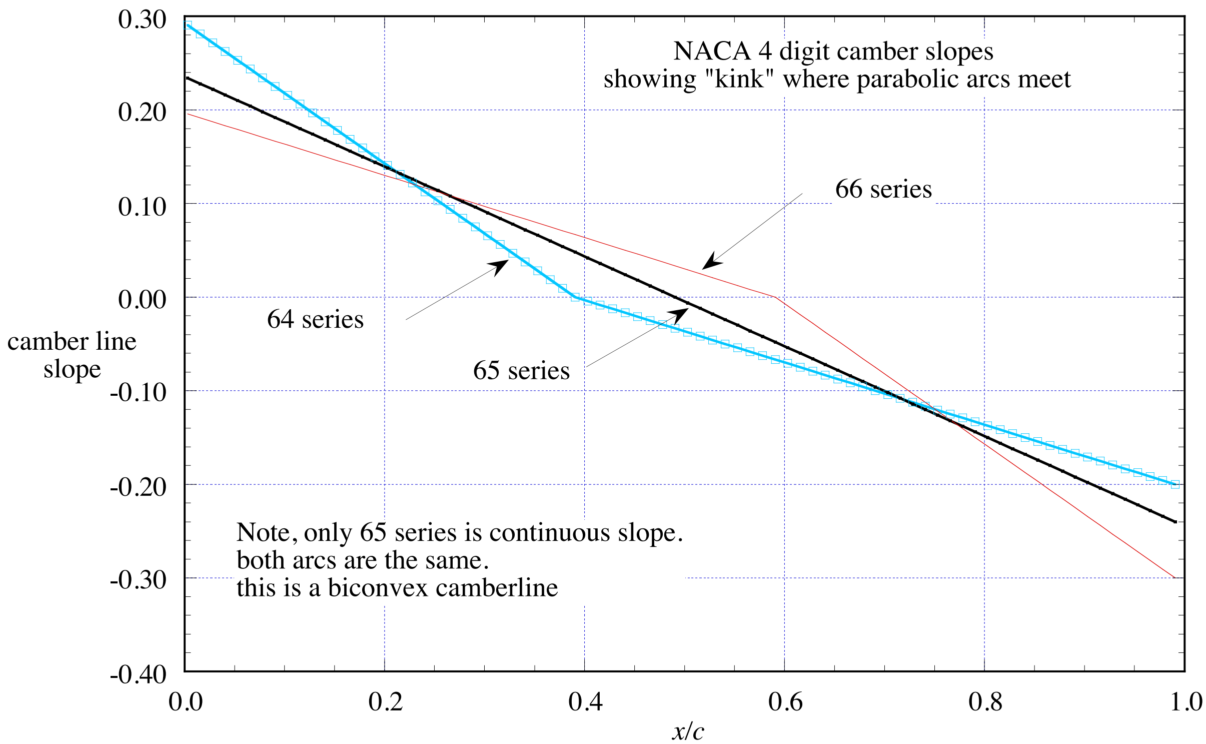

Figure 10. Comparison of camber line slopes for the NACA 4-digit series camberlines 64, 65, 66.

Fig. 11 presents the ΔC_p.'s corresponding to these camber lines. The 65- series camberline corresponds to the biconvex airfoil result. The chordload shows the abrupt change in camber slope for the 64- and 66- series camber lines. Also, the angle of attack for these cases use the values of the ideal angle of attack included with the camber lines in Abbott and Von Doenhoff and they perform as advertized, so that there is no load at the leading edge (the definition of the so-called "ideal" angle of attack). There is an analytical solution for these cases in Riegels, *Aerofoil Sections*, but the equations are lengthy and coding them up will be a task for another day, see *Type* S_2, on page 64 of the English translation. The solution is attributed to Dr. G. Jungclaus.

Concluding comments:

Although the difference between thin airfoil theory and an exact solution shown by McLean surprised me, it's correct. However, McLean also shows the results for an NACA 2405 airfoil. The agreement is better for that airfoil. I think the improved agreement is mainly due to the relatively smaller contribution from the camber line rather than the reduced thickness. *I conclude that the classic camberline contribution from thin airfoil theory is not as accurate as the thickness and flat plate angle of attack components of the theory*. This is definitely a curiosity.

There is probably no point to pursue this analysis further. Although Van Dyke's Report 1274 addresses second order thin airfoil theory, he concentrates on the edge effects, not the lift problem. He presents a way to compute the second order results using tables, but it doesn't look like there's enough detail to develop a computational implementation. I coded up Weber's method years ago, both on a programmable calculator and in BASIC. I don't feel the need to do it again for a modern platform.

In the past people played games with the pressure coefficient formula. Sometimes the linear theory results were improved using a more exact pressure formula. We will not look at that type of ad hoc adjustment here.

I'd never delved in this much detail into thin airfoil theory before. Although it took some work it was interesting. I saw the effect of the "kink" in the 4-digit series airfoil cambers, and it's effect on the chordload. I also found out for myself what the ideal angle of attack meant. Finally, I discovered that some of the methods I hadn't used in years solved these problems instantly on today's computers.

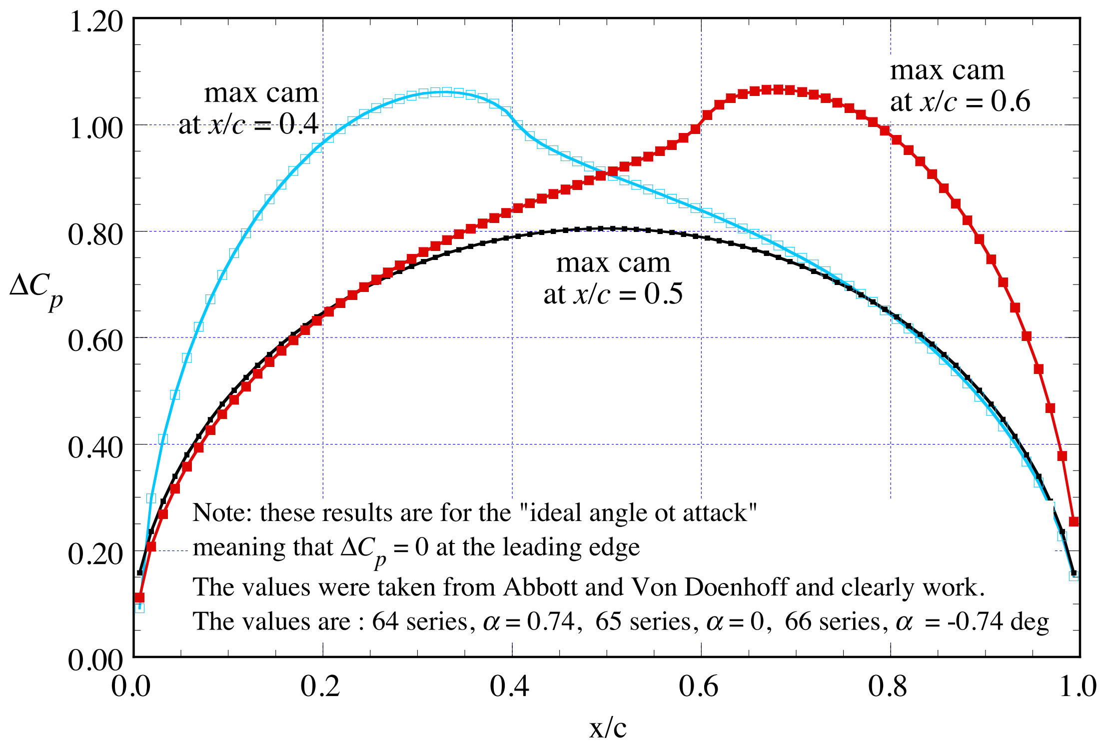

Figure 11. Chordloads for the NACA 4-digit airfoil camber lines at the ideal angle of attack.