W. H. Mason, July 6, 2015/June 14, 2017

**Curiosity Number 8. Airfoil drag (NACA 0012) added to induced drag**

Students get confused about the various contributions to the wing drag polar. The problem stems from the classical 2D airfoil drag polars shown everywhere (and in particular in many basic aerodynamic texts). The 2D airfoil drag polars appear to be parabolic (and as we'll see below for the classic NACA airfoils they are). Note that many modern high performance airfoils don't demonstrate this behavior.

This "curiosity" looks at a simple example to see how including airfoil drag contributes to the classical wing drag polar that is frequently presented as a basic zero lift drag and induced drag. Although I'm not aware of today's basic aero books illustrating this, Nicolai includes it in his aircraft design book (1st edition, Figures 2.7 (C-141) and 2.8 (F-4C), page 2-12, and 2nd edition Figures 2.16 and 2.17 on pages 50 and 51). We will also be including a figure from Warner (1936), and note that Millikan (1941) included this effect. We will plot this slightly differently than they did to bring out the extra airfoil drag effect as clearly as possible.

The NACA 0012 airfoil data that I said was almost parabolic is shown first. Figure 1 illustrates the comparison between a parabola fit to the wind tunnel data at a $C_L$ of 0.66 and the rest of the data in Abbott and von Doenhoff for the Reynolds number 6 million case. I would say the fit is pretty good for our purposes here.

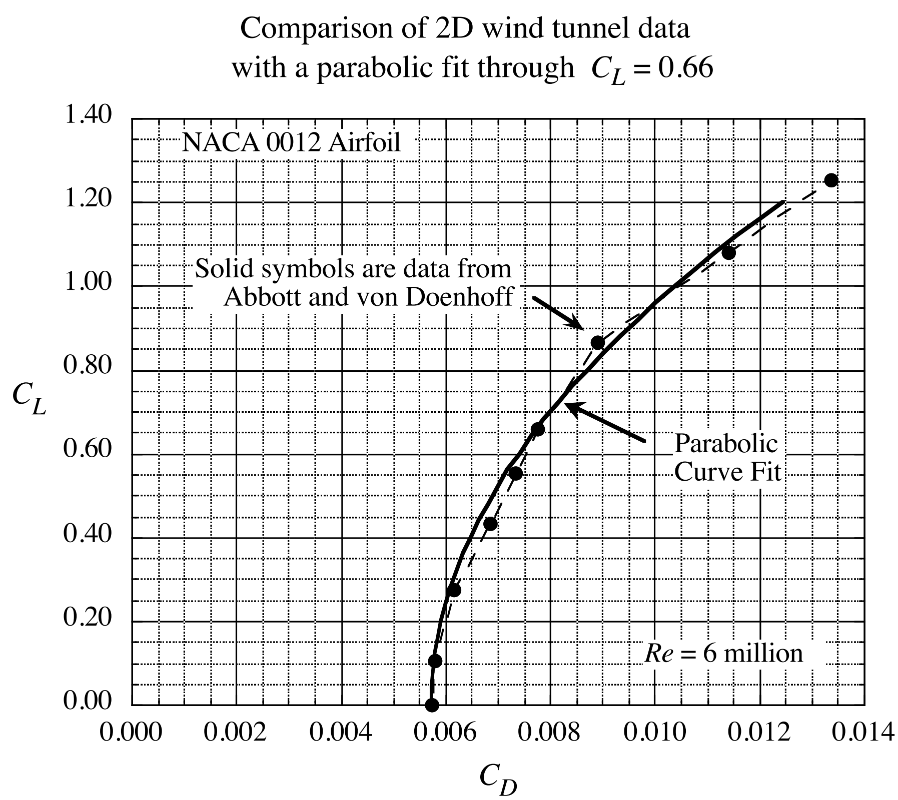


Figure 1. Comparison of NACA 0012 airfoil wing tunnel data with a parabolic curve fit.

Now we'll add the parabolic curve fit for the airfoil data to the induced drag. To make the example as simple as possible we use an untwisted elliptical planform wing and NACA 0012 airfoil wind tunnel data. The use of the elliptical planform results in a constant section $c_l$ across the wing.

The drag, including both the airfoil and induced drag on a symmetrical wing can be found from the following equation as given in Abbott and von Doenhoff, page 10:

$$C_D = \frac{2}{S}\int_0^{b/2} c_d c\, dy + C_{D_i}$$

We can put this in the form I like as:

$$C_D(C_L) = \int_0^1 \frac{c_d(c_l)c}{c_{avg}} d\eta + C_{D_i}$$

It is a good exercise to derive these formulas for yourself. Picking an elliptic planform and recalling that $c_l$ doesn't vary across the span (and in fact $c_l$ and $C_L$ are equal), then $c_d$ is not a function of $\eta$, and can be brought out of the integral. Thus for the elliptic variation of the chord, $c$, and assuming the classical induced drag formula. We find:

$$C_D(C_L) = c_d(c_l) + \frac{C_L^2}{\pi AR}$$

Now, lets break up the airfoil drag into two parts, $c_{d0}$ and a $\Delta c_d$:

$$C_D(C_L) = c_{d_0} + \frac{C_L^2}{\pi AR} + \Delta c_d(c_l)$$

Looking at the data in Abbott and Von Doenhoff we get a value for $c_{d0}$ of 57.3 counts. We will do two cases. Figure 2 shows the drag polar assuming an aspect ratio 6 wing (of course $e$ = 1), and the additional airfoil drag is added above the basic $c_{d0}$ and the induced drag. Figure 3 is for an aspect ratio of 20 wing. This is in part getting ready for C9, a look at E.

We see in Fig. 2 that the airfoil profile drag variation with lift is a relatively small contribution to the total drag polar at low lift, but increases with increasing lift coefficient. Abbott and Von Doenhoff don't include drag data past a $c_l$ of about 1.2, even though they show the maximum lift to be about 1.6. As the lift approaches the maximum, there will be a large airfoil drag contribution to the drag polar because the flow is now separating. We will have to look for other data and show this effect below. This is part of a trend seen in Abbott and Von Doenhoff. Namely they don't examine the airfoil characteristics completely at $c_{lmax}$ and beyond (recall Curiosity No. 3). Note also that it's a little hard to see the airfoil drag contribution, and that's why aerodynamicists often plot $C_D$ - $C_{Di}$ to see the airfoil contribution in a wind tunnel test (they also look at the axial force, but that's another story).

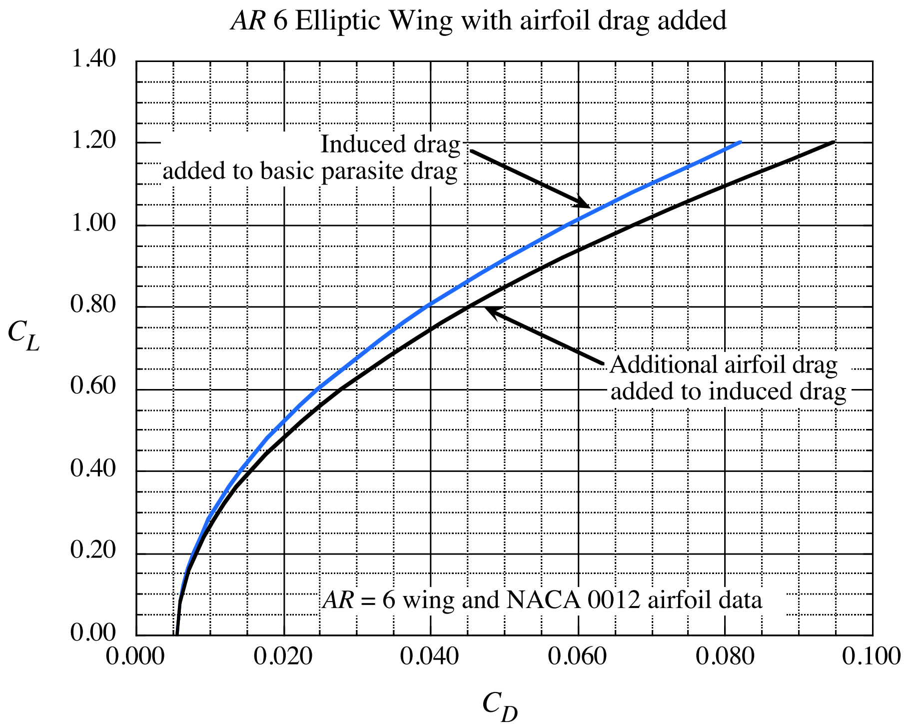


Figure 2. The *AR* = 6 wing drag polar with the NACA 0012 airfoil drag from Abbott and Von Doenhoff added.

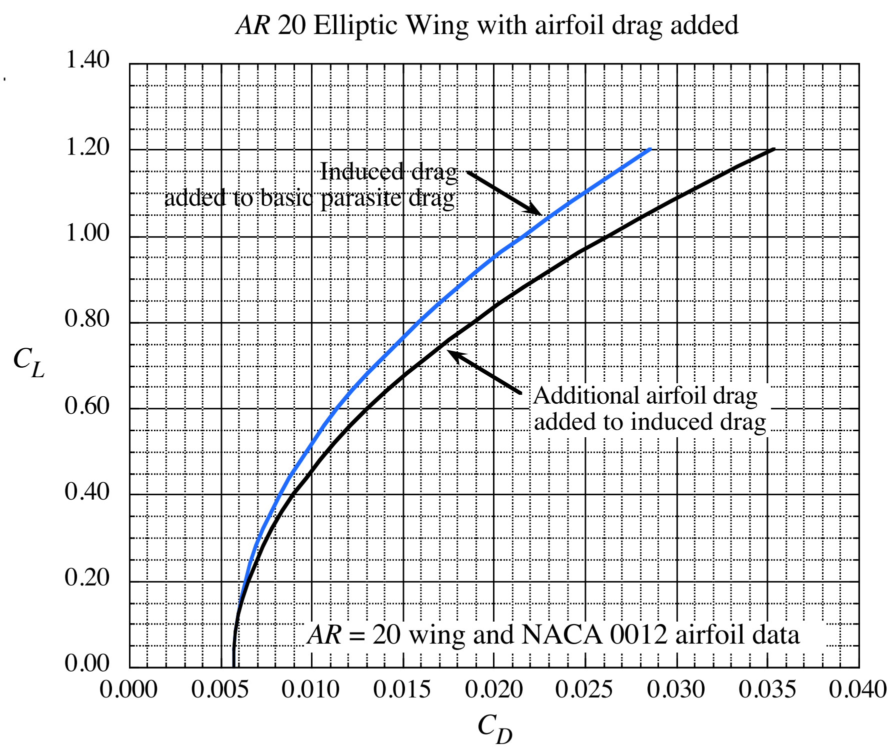


Figure 3. The *AR* = 20 wing drag polar with the NACA 0012 airfoil drag from Abbott and Von Doenhoff added. Note difference in $C_D$ scale compared to Fig. 2.

Clearly as the aspect ratio increases the contribution of the additional airfoil drag with lift becomes a larger percentage of the drag due to lift.

Upon looking at these figures I wondered about other similar plots. Eventually I recalled a figure in Edward Warner's *Airplane Design – Performance* book (McGraw-Hill, 1936).[1] For subsonic aerodynamics this book is one that every aerodynamicist should have read. My copy is falling apart and I don't lend it out anymore. Although not plotted in exactly the same way, Fig. 4 from Warner (Fig. 96, pg 166) gives me some confidence that the effects in Fig. 2 and 3 are correct. Note that Millikan, *Aerodynamics of the Airplane* (Wiley, 1941) has essentially the same figure as Warner (Millikan, Fig. 1-44, pg 52), but doesn't provide the wing aspect ratio or specific airfoil section data used.

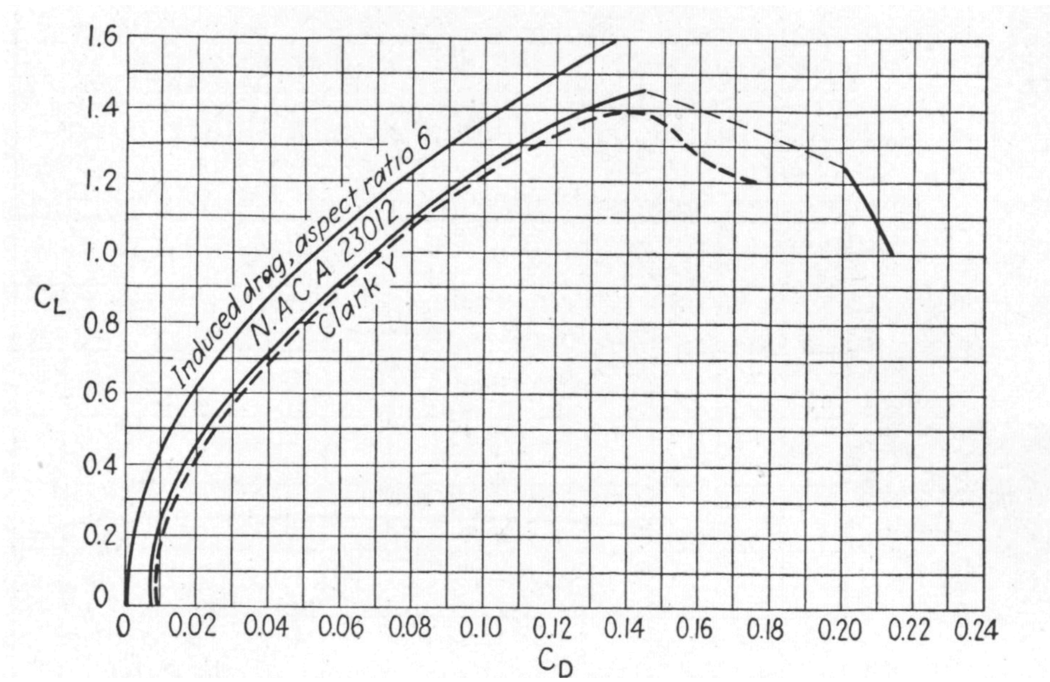


Figure 4. Warner's airfoil and induced drag polar (Fig. 96, pg 166)

Doug McLean makes an issue of making a mistake by ignoring the airfoil drag when estimating the $C_L$ for $L/D_{max}$. (*Understanding Aerodynamics*). This is certainly true when transonic aerodynamic design is pushing the limits of wing/airfoil performance. Lets look at the quantitative effect for low subsonic Mach numbers. Since we used a parabolic curve fit for the airfoil drag we can do this problem analytically. Using Abbott and Von Doenhoff data, the $C_{D0}$ is 0.00573.

| | $AR = 6$ | $AR = 20$ |
|---|---|---|
| $C_{L\ L/Dmax}$ without additional airfoil drag | 0.329 | 0.600 |
| $C_{L\ L/Dmax}$ with additional airfoil drag | 0.315 | 0.527 |
| $L/D_{max}$ without additional airfoil drag | 28.68 | 52.36 |
| $L/D_{max}$ with additional airfoil drag | 27.49 | 46.04 |

[1] Checking Amazon just now, some are available for $12.50 or so. A super deal.

So including the additional airfoil drag with lift does affect the values, and is larger for higher aspect ratios, where the airfoil drag becomes a larger percentage of the drag due to lift. Steve Brandt tipped me off to this effect. Nicolai includes formulas that address camber.

Since we'd like to look closer to $C_{Lmax}$, we looked for some other data. We found two other "two dimensional" NACA 0012 airfoil data sets that were easy to examine. One was the ONERA data in AGARD AR-138, intended to be used for CFD code validation. The other data was contained in NASA TM 4074 by Ladson, also to be used for code validation. The measured lift coefficients as a function of angle of attack are shown in Fig. 5. Considering that many aerodynamicists assume that we should be able to do these measurements with little uncertainty, it is surprising that we don't get better agreement.

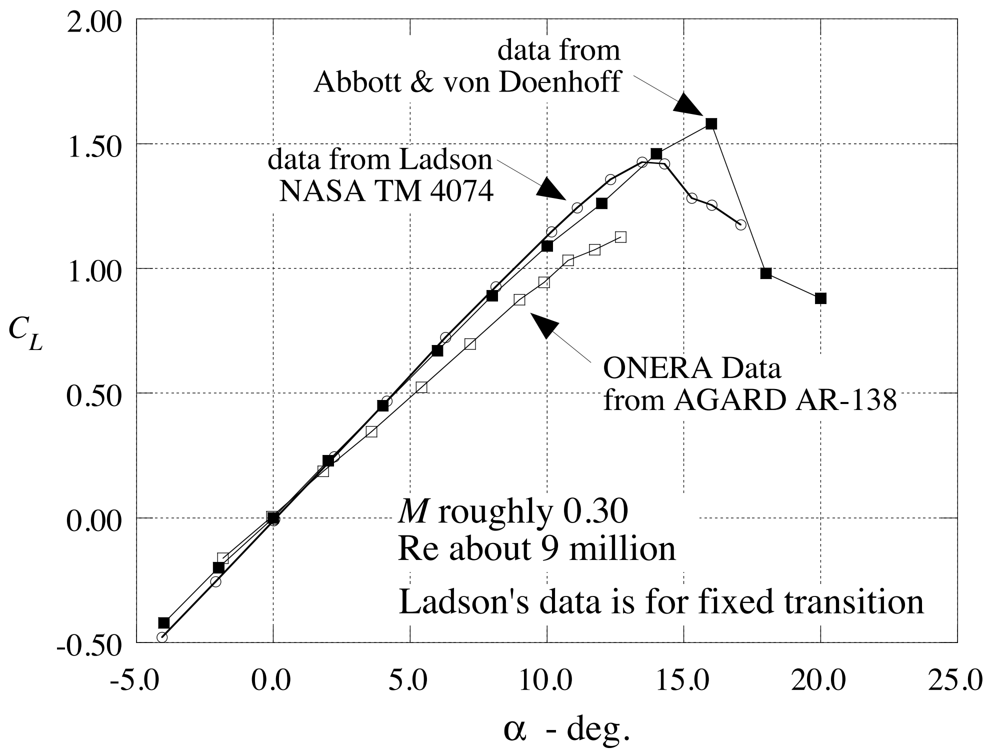


Figure 5. Comparison of three different sets of wind tunnel tests for the NACA 0012 airfoil at low speed.

What are we to make of this? The NACA/NASA Langley data is in good agreement up to a $C_L$ of about 1. However, both the $C_{Lmax}$ and the character of the stall differs between the two tests. The data from ONERA (France) has a fundamentally different slope and seems to be stalling before the Langley results.

Once again results we typically take for granted give us pause. Am I the only one this bothers? I haven't seen these problems raised in public. I have been slightly disappointed with the progress made in the Drag Prediction Workshops, but those problems are a lot harder than a 2D subsonic airfoil case.

We repeat Fig. 1 using Ladson's airfoil data. Since he tested the airfoil over a wide variety of conditions, we look at the airfoil drag results at $M$ = 0.3 and an $Re$ of 9 million. We look at three different cases for transition: free, 60-w and 120 grit. The 60-w indicates a "wraparound the leading edge up to 5% chord" case, as was apparently done in the NACA tests using a transition strip.

Figure 6 shows the drag polar results for the three cases. Notice that the minimum drag doesn't occur at a zero lift coefficient. Also note the large penalty in both drag and lift using the 60 grit wrapped all the way around the leading edge. There is also a drag penalty associated with the 120 grit case, however the general shape of the drag polar is similar to the free transition case. We will use the 120 grit data to make the wing drag polar including the airfoil drag. That's because it seems to be smoothest around zero lift. Curiously, the minimum drag is never below 60 counts (recall that Abbott and von Doenhoff results give a minimum value of 57.3 counts. Ladson's report should be downloaded and examined for the trends in Mach number and Reynolds number. There's lots of food for thought there.

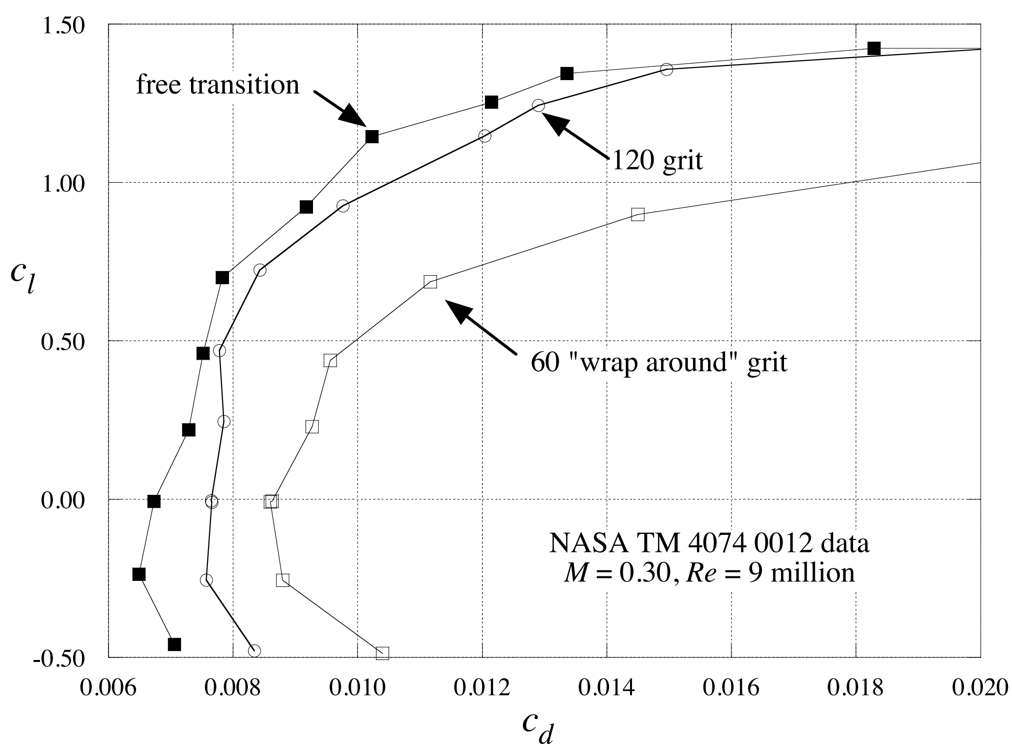


Figure 6. NASA TM 4074 WT Test of the NACA 0012 airfoil.

Figure 7 is the resulting drag polar for the elliptical *AR* = 6 wing with Ladson's NACA 0012 airfoil data included. It is interesting to see that the airfoil drag doesn't impact the polar significantly until the airfoil stalls. That's when the drag due to viscous effects increases dramatically. I did this case because I thought the effect would be larger leading up to $C_{Lmax}$. Because we include the large drag at $C_{Lmax}$, the axis is significantly larger than the one we were previously using, and the additional viscous airfoil drag appears to make a very small addition to the drag [polar.

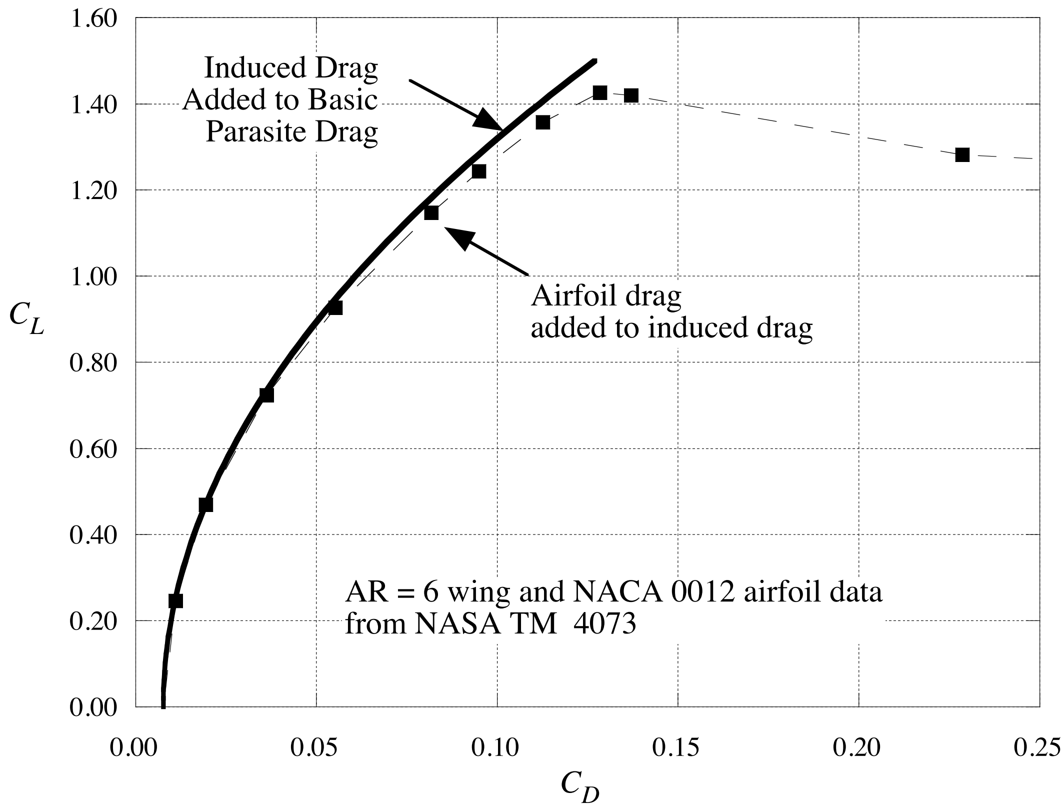


Figure 7. Drag polar for the *AR* = 6 elliptical wing with NACA airfoil data, going all the way to $C_{Lmax}$. The airfoil drag is from NASA TM 4073, and the transition strip of 120 case was selected (as shown in Fig. 6).

*What are the takeaways from this curiosity?*

- Students need to understand the difference between additional airfoil drag with lift and induced drag. With a good airfoil the additional drag should be small.
- We should not ignore additional airfoil drag when creating a polar.
- There is non-negligible uncertainty in the basic airfoil data from wind tunnel tests, even at low Mach numbers. This is unsettling even to me.

End note: if you are feeling brave look at McCroskey's evaluation of NACA 0012 wind tunnel results, NASA TM 100019, 1987. It's not clear to me that there's been any significant reduction in uncertainty since that work.