

# F-35: A case study

Eugene Heim  
Leifur Thor Leifsson  
Evan Neblett

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Course: AOE 4124 Configuration Aerodynamics

Project: A case study of the F-35 JSF

Instructor: Dr. W.H. Mason, professor

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Presenters: Eugene Heim, Evan Neblett and Leifur Thor Leifsson

# Outline

- ✦ F-35 Geometry
- ✦ VLMpc/VLM4998
  - ✦ Geometry models
  - ✦ Planform analysis
- ✦ Friction
  - ✦ Best subsonic cruise condition
  - ✦ Skin friction drag
- ✦ LamDes2
  - ✦ Trimmed performance and load split
  - ✦ Optimal twist distribution
- ✦ TSFOIL22
  - ✦ Transonic airfoil performance
- ✦ Comparison of Configurations
- ✦ Summary

# F-35 Geometry

F-35A/B CTOL/STOVL



F-35C CV



	F-35 A/B	F-35 C
b (ft)	35.10	43.50
l (ft)	50.75	51.20
h (ft)	15.00	15.50
$S_{Ref}$ (ft <sup>2</sup> )	460.0	620.0
AR	2.68	3.05
$c_{av}$ (ft)	13.12	14.24
GTOW (lbs)	50,000	57,000



Figures from and numbers from [4].

# Planform Analysis

- The analysis gives
  - Longitudinal derivatives
  - Neutral point
  - Static margin
- Planforms analyzed in
  - VLMpc: 2 sections
  - VLM4998: 4 sections
- Difficult to get a uniform grid in VLMpc for these configurations
- A “better” grid obtained in VLM4998
- Little difference in results!

- The purpose of the planform analysis is to get: longitudinal derivatives ( $C_{L\alpha}$  and  $C_{M\alpha}$ ), neutral point (is the aerodynamic center ( $dC_m/dC_L=0$ ) of the whole configuration) and static margin ( $SM = x_{cg}/c_A - x_{np}/c_A$ , where  $x_{cg}$  is center of gravity,  $x_{np}$  is the neutral point location and  $c_A$  is the average chord length).
- Planform were analyzed in VLMpc and VLM4998.
- VLM4998 is able to handle 4 planforms and therefore can give a better representation of the planform. VLMpc on the otherhand can only handle 2 planforms.
- A “better” grid means that the control point locations are more smooth and should represent the planform better. However, we found that the there was not much difference in the results between the two.

## Planform Models

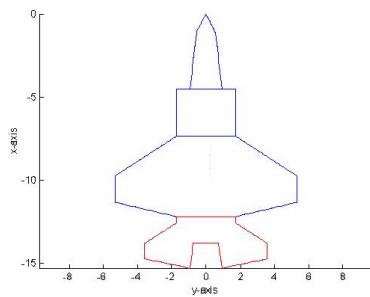
Picture from [5]

Planform model used for VLM analysis

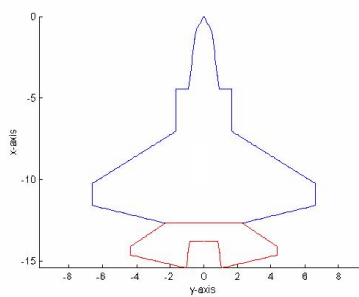
- Simplified to straight and parallel lines
- Streamwise segments parallel to flow
- Wing and tail modeled as separate planforms



F-35A/B CTOL/STOVL



F-35C CV

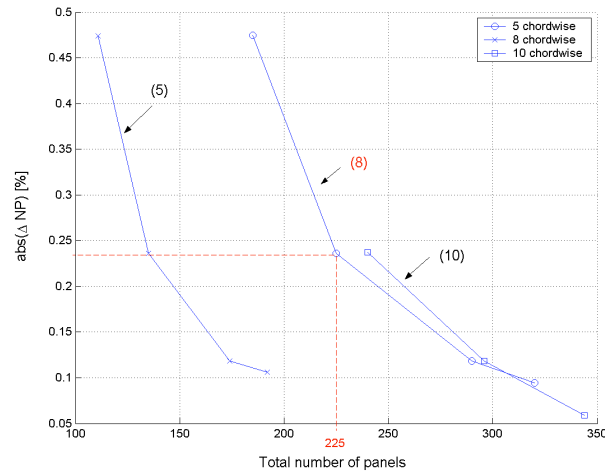


Dimensions in meters.

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- Vertex points pulled over aircraft top view image with MatLab and scaled to full size using the wing span.
- All curved surfaces simplified to straight lines (ie nose)
- Highly swept line segments were forced parallel with the streamwise flow
- F-35A/B
  - 2-section planform used for VLMpc and 4-section planform used in VLM4998
- F-35C
  - 2-section planform used in VLMpc. No VLM4998

# Grid Convergence

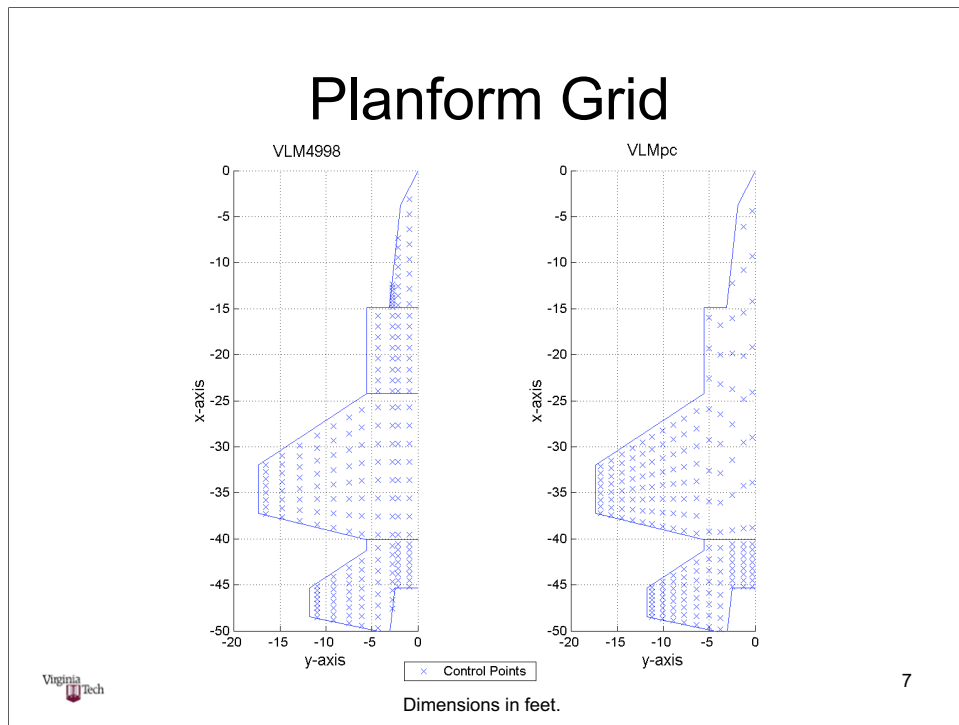


Grid selected: 8 chordwise panels per section and total of 225 panels

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- This graph shows the absolute change in neutral point with total number of panels.
- The number of chordwise panels is the number used for each section of the planform.
- These values were obtained by finding the location of center of gravity that gave a static margin of zero. This was done by manual iteration of VLMpc and the convergence criteria used was  $\text{abs}(0.001)$  value of the static margin.
- This study was performed for chordwise panels of 5, 8 and 10 per section.
- We see that even using 5 chordwise panels per section and a total of 115 panels gave a small change in the neutral point location.
- We also that how the curves converge rapidly.
- It was however fairly difficult to do the convergence test when using 10 chordwise panels. The convergence criteria used was then higher than the changes obtained.
- From all of this and from visualizing the grid (control point locations) the grid selected was 8 chordwise panels per section and a total of 225 panels.

# Planform Grid



- This slide shows the planform grid obtained by VLM4998 and VLMpc. The grid for VLM4998 has 225 control points while the VLMpc grid has 200. The difference is that the programs automatically adjust the number of control points per section when generating the grid.
- The “x” marks the location of the control points.
- VLM4998 allows the user to use 4 sections but VLMpc only 2.
- This can result in a “better” grid, meaning that the control point location is more uniform, giving a better representation of the planform.
- When setting up the planform the edges were made streamwise. Spanwise breaks in the planform are lined up on common breaks. This helped in making the grid more uniform.

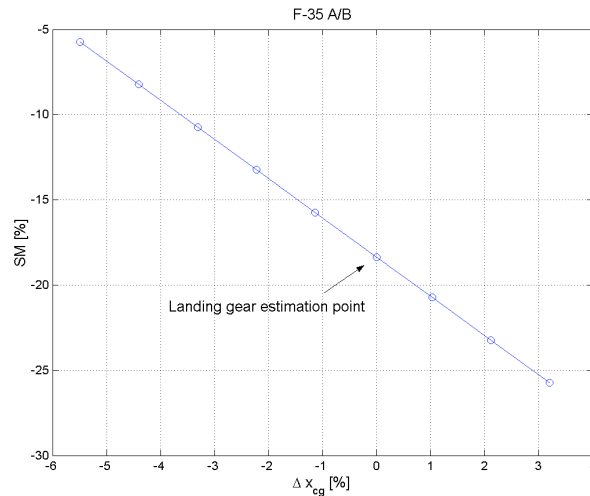
# Planform Analysis Results

	F-35 A/B	F-35 C
Code utilized	VLMpc	VLMpc
$C_{L_2}$ ( $\text{rad}^{-1}$ )	3.78	3.55
$C_{M_2}$ ( $\text{rad}^{-1}$ )	0.69	0.35
$x_{np}$ (ft)	27.67	29.10
$x_{cg}$ (ft)	30.53	30.53
SM [%]	-19.3	-9.6

- This slide shows the results of the planform analysis for F-35 A/B (done using VLM4998) and F-35 C (done using VLMpc).
- The F-35 A/B configuration was also analyzed in VLMpc but there was almost no difference in the results and therefore they are not shown here.
- The center of gravity location,  $x_{cg}$ , was obtained from measurements of pictures obtained at the Lockheed Martin website. It was done by finding the location where the angle from the aft landing gear to the cg of the aircraft is 15 degrees. This is a rule of thumb and therefore is not very accurate, but it should get you in the ballpark. The cg was assumed to be the same for the both configurations.
- The A/B configuration is estimated to be 19.3% unstable and the C configuration 9.6% unstable.
- The static margin estimation depends strongly on the cg estimation. Therefore any error originated from the cg estimation would also effect the static margin estimation. As said here above, the cg estimation isn't very accurate. So, these values should be evaluated with that in mind.
- Next slide shows the static margin sensitivity to cg location.



# Static Margin Sensitivity



- This slide shows the sensitivity of the static margin to the location of center of gravity for the F-35 A/B configuration.
- The landing gear estimation point is the reference point here.
- These results were found by using VLMpc.
- We see how the static margin is highly dependent of the center of gravity location. By moving the cg forward by around 4.5% the static margin changes from around -19% to -8%. That corresponds to around 1.3 feet of change in cg location.

# Best Subsonic Cruise

1 of 2

Symbol	Value	Description
$\kappa_A$	0.91	Airfoil technology factor
$t/c$	0.06	Thickness ratio
$C_l$	0.26	Section lift coefficient
$\alpha_{c/4}$ (deg)	23.4	Quarter chord sweep
$M_{DD}$	0.89	Drag divergence Mach number
$M_{crit}$	0.78	Critical Mach number
<b><math>M_{cruise}</math></b>	<b>0.75</b>	<b>Cruise Mach number</b>

$$M_{DD} = \frac{\kappa_A}{\cos \Lambda} - \frac{t/c}{\cos^2 \Lambda} - \frac{C_l}{10 \cos^3 \Lambda} \quad M_{crit} = M_{DD} - \left(\frac{0.1}{80}\right)^{1/3}$$



Equations from Mason [2].

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- This slide shows how we selected the cruise mach number. We based our selection on calculations of the drag divergence mach number and the corresponding critical mach number. The equations used are shown below the table and they are also given at the back of the presentation with definitions of the symbols. These equations are from Mason [2]. Let's look at the table:
- The technology factor was set to 0.91, because the airfoil we used is a modified 6-series. This factor should be 0.87 for a unmodified 6-series airfoil and 0.95 for a supercritical airfoil. Thus, we set this factor to the mean of those.
- The thickness ratio was set to 0.06, corresponding to a 6% thick airfoil. The airfoils on the F-16 are around 4.5% thick and around 6% thick for the F-22. We think that the airfoils on the F-35 are a little bit thicker than those of F-16, probably similar to F-22, because then its possible to make the aircraft lighter and the wing can carry more fuel.
- The section lift coefficient was found from the planform analysis, i.e. from VLMpc and VLM4998. It was set to 0.26, which is the lowest section lift coefficient along the span. Thus, our estimation of the drag divergence number will be rather conservative.
- The sweep angle was set to the quarter chord sweep.
- The drag divergence number was calculated to be 0.89 and the critical mach number was then found to be 0.78.
- So, to make sure that there are shock waves causing wave drag we selected the cruise mach number to be 0.75.
- This value is a conservative estimate and it is most likely that the F-35 will cruise at a higher value, perhaps around 0.8 or 0.85.

# Best Subsonic Cruise

2 of 2

➔ At cruise condition  $M = 0.75$

	F-35 A/B	F-35 C
$C_{D0}$	0.0095	0.010
$e$	0.995	0.995
$(L/D)_{max}$	15.25	15.44
$C_{L@}(L/D)_{max}$	0.275	0.310
$h_{cruise}$ (ft)	19,000	25,600

$$\left(\frac{L}{D}\right)_{MAX} = \frac{1}{2} \sqrt{\frac{\pi A R e}{C_{D0}}} \quad C_{L,(L/D)_{max}} = \sqrt{\pi A R e C_{D0}} \quad P = \frac{2W}{\gamma M^2 C_{L,(L/D)_{max}} S_R}$$

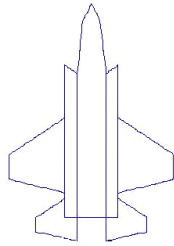
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- This slide shows our estimates at the selected cruise mach number. Again, the equations used are showed below the table, but they are defined at the end of the presentation.
- These equations are obtained from setting the skin friction drag equal to the induced drag. Giving the maximum value of lift to drag and the corresponding lift coefficient. We are assuming that there is no drag due to shocks, i.e. wave drag and whole flowfield is subsonic.
- The cruise altitude to satisfy these conditions was found by calculating the air pressure and finding the corresponding altitude from a table for a standard atmosphere in [1]. The air pressure is found from the definition of lift coefficient and writing the velocity as a function of mach number and the speed of sound. In addition to that assuming a perfect gas and writing the speed of sound as a function of pressure and density.
- These values are a conservative estimate because they depend on the estimate of the cruise mach number.
- Comparing these values to other fighter jets, we see that the .....

# Skin Friction Analysis 1 of 2

- “Friction” code utilized to obtain skin friction and form drag
- Verified with F-15 sample input/output [6]
- 10 section model used

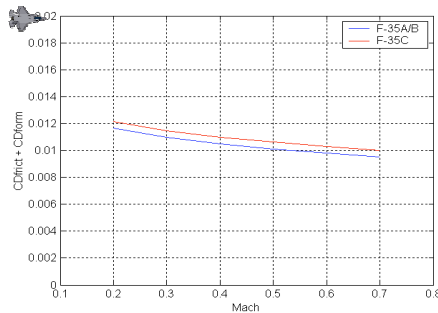


F-35A/B Friction model

	F-35A/B	F-35C
Component	$S_{wet}$ (ft <sup>2</sup> )	$S_{wet}$ (ft <sup>2</sup> )
CANOPY	75	75
FUSELAGE	350	380
NACELLES (2)	510	520
OUTB'D WING (2)	440	640
HORIZ. TAIL (2)	125	215
VERT TAIL (2)	100	100
<b>TOTAL</b>	<b>1600</b>	<b>1930</b>

- The “Friction” code was utilized to obtain estimates of the skin friction drag and form drag. These results provided an estimate of the Cdo of the aircraft.
- The aircraft was modeled with 10 components for inputting into the program:
  - Canopy – Half and ellipsoid body
  - Fuselage – Cylinder and a Cone
  - Wings (2) – Thin plates
  - Nacelles (2) – Rectangular Prism
  - Horizontal Tails (2) – Thin plates
  - Vertical Tails (2) – Thin plates
- An image of the aircraft was measured and area were pulled off and scaled up to full size

## Skin Friction Analysis 2 of 2



→  $C_{D_0}$  values estimated at  $M_{cruise}$

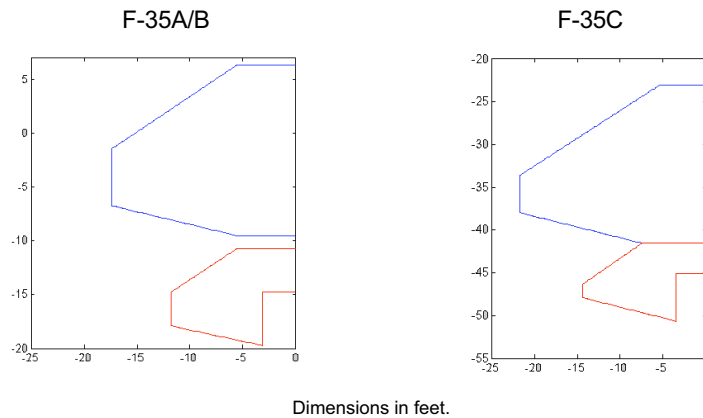
→ **F-35A/B –  $C_{D_0} = 0.0095$**

→ **F-35C –  $C_{D_0} = 0.0100$**

- The sum of the skin friction drag and form drag were found for a range of subsonic Mach numbers.
- The value of the sum near the cruise Mach number was used as an estimate of the  $C_{D_0}$  of the aircraft.
- The  $C_{D_0}$  for the F-35C was slightly higher due to the enlarged wing and tail area. A slight increase in skin friction drag.

# Trimmed Performance

- ✦ LamDes utilized
- ✦ Verified with forward swept wing sample input/output [6]
- ✦ Planform of flight surfaces only



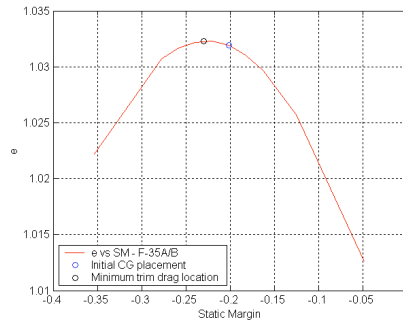
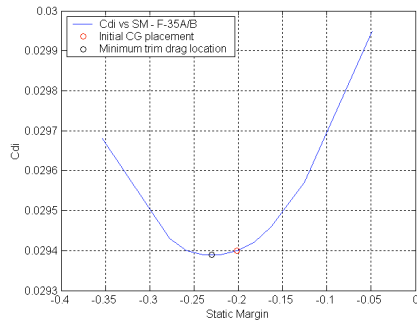
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- LamDes was used to analyze the performance of the aircraft with a varying CG location.
- The sample input for a forward swept wing canard configuration was used to verify the program by comparing the experimental output with the known output
- Many different planforms for each aircraft were ran in LamDes. The aerodynamic values outputted varied little with change in planform. A planform consisting of only the flight surfaces was chosen to analyze to reduce the chance of invalid data of wing twist.

## Trimmed Performance – F-35A/B

Code run for varying CG location of  $\pm 2$  ft

Cdi plotted against Static Margin



F-35A/B trimmed performance

$C_{di} = 0.02939$      $e = 1.0325$

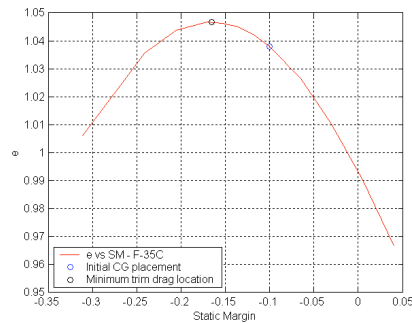
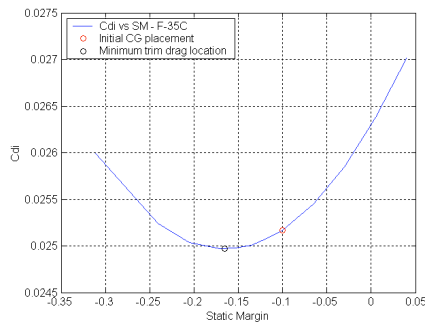
@ 23% Unstable!



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- To analyze the trimmed performance, LamDes was run for a range of CG locations fore and aft of the initial CG guess from the rule-of-thumb for main landing gear placement.
- The  $C_{di}$  was plotted against static margin to find the CG location for minimum induced drag.
- The efficiency ( $e$ ) was also plotted against the static margin for the maximum efficiency location of the planform.
- For the A/B variants the min drag location was close to our initial estimate of the CG location.
- The stability for the A/B variants at minimum trim drag is 23% unstable

## Trimmed Performance – F-35C



### F-35C trimmed performance

$$C_{di} = 0.0249 \quad e = 1.046$$

@ 17% Unstable!



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- To analyze the trimmed performance, LamDes was run for a range of CG locations fore and aft of the initial CG guess from the rule-of-thumb for main landing gear placement.
- The  $C_{di}$  was plotted against static margin to find the CG location for minimum induced drag.
- The efficiency ( $e$ ) was also plotted against the static margin for the maximum efficiency location of the planform.
- For the A/B variants the min drag location was close to our initial estimate of the CG location.
- The stability for the A/B variants at minimum trim drag is 17% unstable



# Wing/Tail Load Split

✈ Results from LamDes

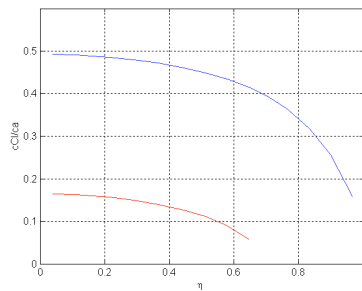
✈ Evaluated at  $C_{l_{des}} = 0.5$ ,  $M = 0.75$  @ min trim drag CG

**F-35A/B**

Wing  $C_l = 0.4122$

Tail  $C_l = 0.0894$

**82%/18% Wing/Tail**

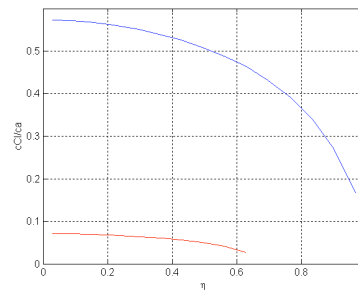


**F-35C**

Wing  $C_l = 0.4247$

Tail  $C_l = 0.0753$

**85%/15% Wing/Tail**



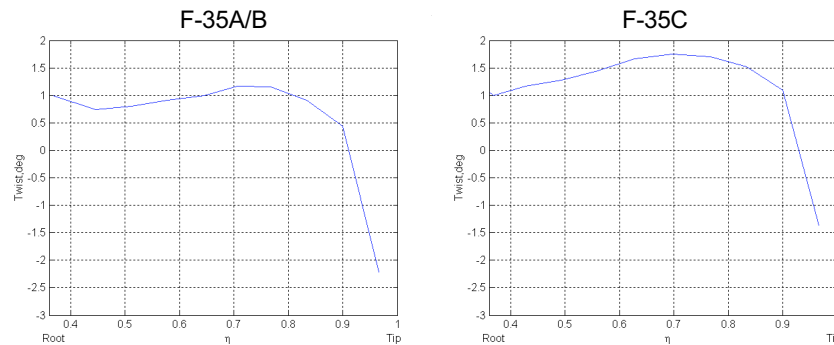
Wing  
Tail

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- LamDes also provides the lift coefficients for each planform. Dividing these  $C_l$ 's by the design  $C_l$  inputted gives the load each surface carries as a percent of the total.
- The load was computed at the min trim drag cg location
- As the CG is varied the load each surface pick up will vary linearly increasing or decreasing depending on which way the CG shifts
- Plotting the  $cCl/ca$  of each surface vs  $\eta$  shows the spanload of each surface and the load each picks up.

# Wing Twist

- ✈ Vertical offset required to get reasonable twist results from LamDes
- ✈ Root Incidence of wing assumed to be 1 deg



- LamDes provides the twist for both planforms inputted. The wing twist was computed for Clides of 0.5, Mach of 0.75, at the minimum trim drag CG location.
- To get reasonable twist values an offset in height of the flight surfaces was required. A vertical offset of 2 feet was used to obtain the twist values. Decreasing the vertical offset much lower than 2 feet resulted in sketchy twist distributions.
- For consideration when plotting the root incidence of the wing was assumed to be 1 deg.
- The plots show the twist from the root of the wing to the tip. The twist for the portion of the wing inside of the was neglected as it is irrelevant.

## Transonic airfoil performance

- Lockheed Martin's air dominance airfoil [5]...
  - F-16 (4%)
  - F-22 (5.92% to 4.29%)
  - JSF...thicker?
    - Improved transonic performance
    - Structurally more efficient
    - Increased internal fuel volume
  
- We chose a 6% 64-A-206

We could not find the airfoil that was used on the F-35 or even if there is more than one airfoil. So we looked at airfoils used in the past by Lockheed Martin for their air superiority aircraft. We found the F-16 and the recent F-22 both use versions of the 64-A-2 series. Looking at the increasing trend in airfoil thickness and keeping in mind performance as well as the underlying goal of the JSF program to reduce cost, we choose a 6% 64-A-206. This thicker airfoil allows for improved transonic performance, it is structurally more efficient, and has more internal fuel volume.

# Numerical Tools

## ➤ TSFOIL

- Transonic small disturbance theory
- Modified by “Dr.” Andy Ko and Dr. Mason

## ➤ Matlab - Sidestepping Legacy Code

- TSFOIL automation
- Post processing of output

The numerical tools we used were a modified version of TSFOIL and Matlab. TSFOIL was used to solve the transonic small disturbance theory equations. While Matlab was used to sidestep the mundane tasks of the TSFOIL legacy code.

# TSFOIL

- Jameson type input
  - Modified SC20610.inp sample input
  - AOA, t/c, Mach, Iterations, Coordinates.
  
- Output file investigation...(not in manual)
  - Multiple mesh sizes...Convergence
  - Force and moment coefficient
  - Cp distribution
  - Mach Map

We modified the necessary variables listed from the sample Jameson type input file. There was no description of the output in the TSFOIL manual so we had to decipher what everything meant. We found that the code numerically solves the TSDT equations for a coarse grid, medium grid and final grid pending the solutions don't diverge. It give the lift and pitching moment coefficients, Cp distribution for each grid and a Mach Map for the last grid that didn't diverge. Finally the it solves the momentum integral for an inviscid estimation of the drag and finds the wave drag if there are shocks present

## Matlab - Sidestepping Legacy Code

- Automating TSFOIL
  - Input file generation and piping
  - Output file processing
    - Coefficient contours
    - Convergence history of final mesh
    - Cp distributions
    - Colorized Mach maps
    - Airfoil AOA animation

We used the versatility of Matlab to automate the input file generation and then used the dos command in Matlab to pipe the input file into TSFOIL. TSFOIL had to be slightly modified by removing all pause statements in the code. Once the TSFOIL was finished the output file was interrogated for known anchor points from which the data could be referenced and read in from by using textread in Matlab. With the process automated we could construct a matrix of lift, drag, and moment coefficients versus angle-of-attack and Mach number, a plot of the convergence history, pressure coefficient distributions, and we could colorize the Mach Maps. We even added an airfoil angle-of-attack animation for your viewing pleasure.

## 2D Analysis

### ➤ 3D to 2D conversion

➤ 3D cruise condition, Mach = 0.75, t/c = 6%

➤ 2D Mach number, Mach = 0.69, t/c = 6.5%

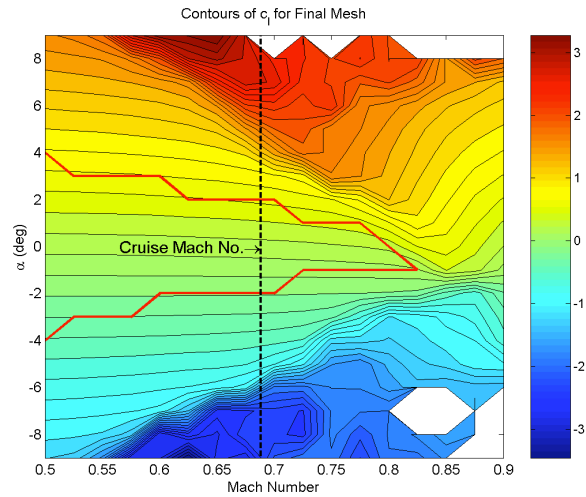
### ➤ TSFOIL limits

➤ Mach 0.5 to 2.0...(0.9)

➤ Angle-of-attack  $-9^\circ$  to  $+9^\circ$

The 3D cruise Mach number gave a 2D design Mach number of 0.69 and an increase in thickness of 0.5%. We tested this airfoil over the limits of TSFOIL. If you entered a Mach number below 0.5 in the input file TSFOIL would return the limits for Mach number are 0.5 to 2.0 in the output file. However, in the version we used, if a Mach number of 1.0 or higher was inputted TSFOIL would crash.

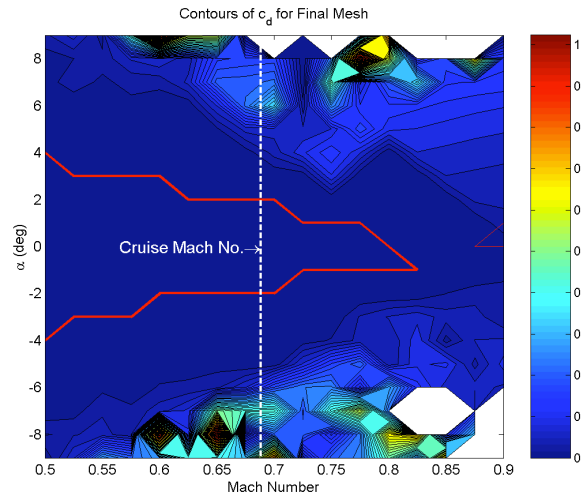
# Lift Coefficient Contour



Here is the resulting lift coefficient contour for the final mesh. The white areas are where the final mesh diverged. The area inside the red line is converged data whereas everywhere else the iteration limits were reached. You can see at our 2D design Mach number the data converges from  $\pm 2^\circ$ .

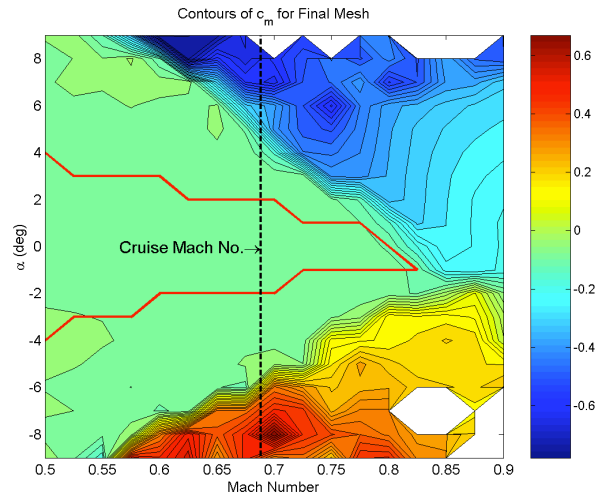


# Drag Coefficient Contour



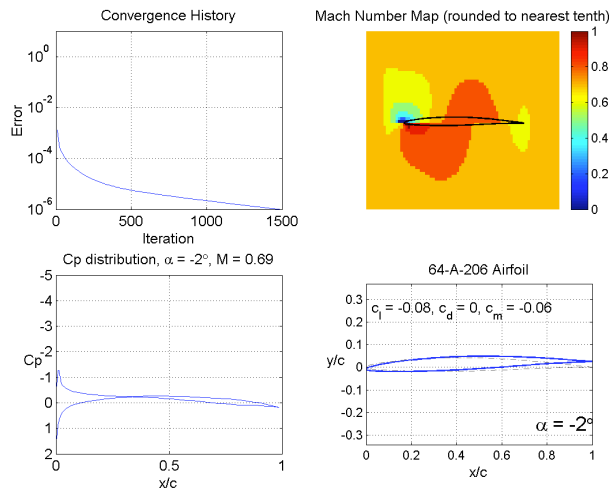
There are a few points on this graph that distort the variance of CD. However the drag calculations are for inviscid flow and are very small.

## Pitching Moment Coefficient Contour



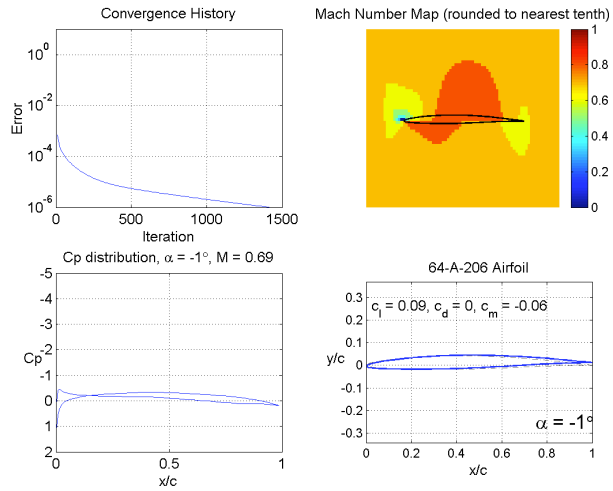
Likewise the pitching moment distribution appears pretty well constant for converged results. We found that whenever a shock formed on the airfoil the data tended not to converge.

# Converged Cruise Conditions



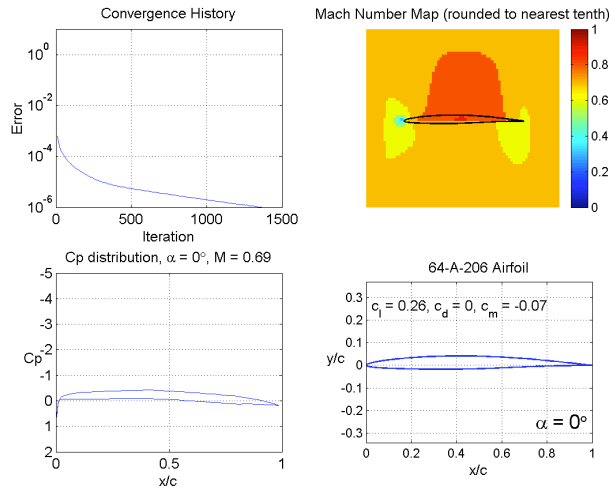
Here is the output of TSFOIL for converged cruise conditions straight from the Matlab code. This plot is for -2 degrees. In the upper left you can see the data converged around 1500 iterations. The lower left shows the cp distributions, the upper right is a colorized version of the Mach Map and in the lower right you can see the coefficients and angle-of-attack of the airfoil.

# Converged Cruise Conditions



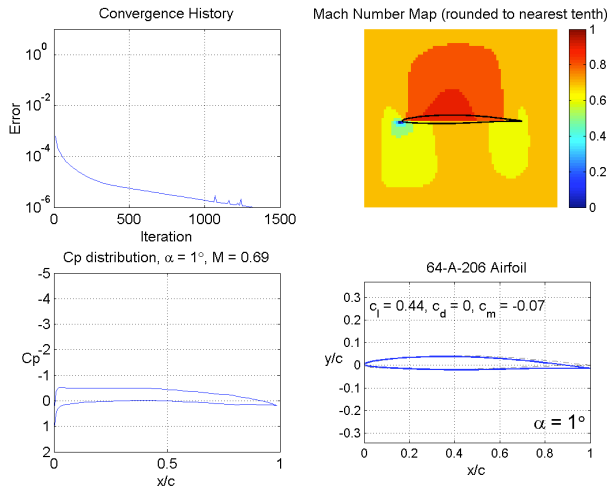
-1 degrees

# Converged Cruise Conditions



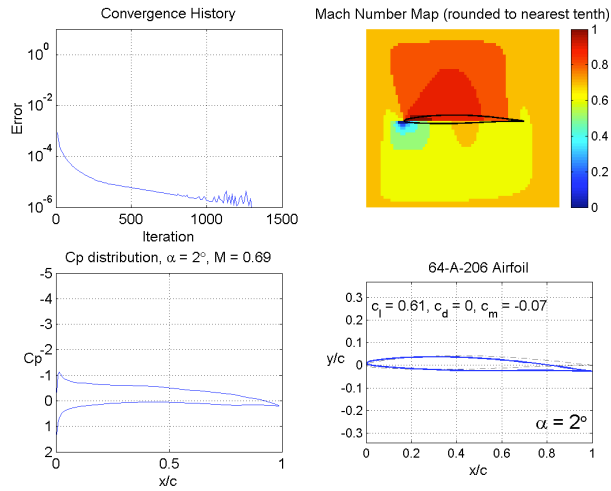
0 degrees

# Converged Cruise Conditions



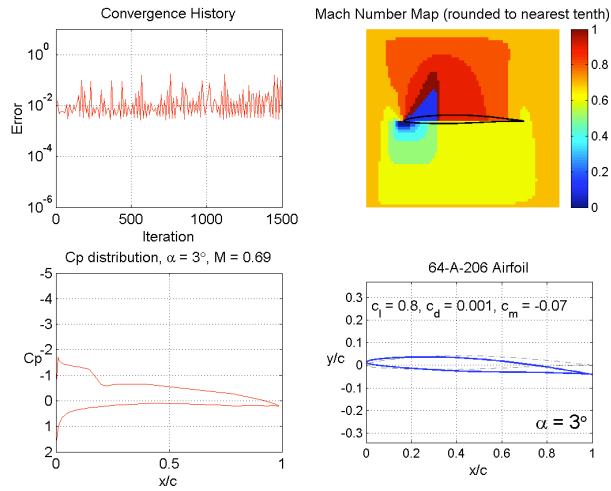
1 degree

# Converged Cruise Conditions



And 2 degrees. Notice the velocity distributions around the airfoil.

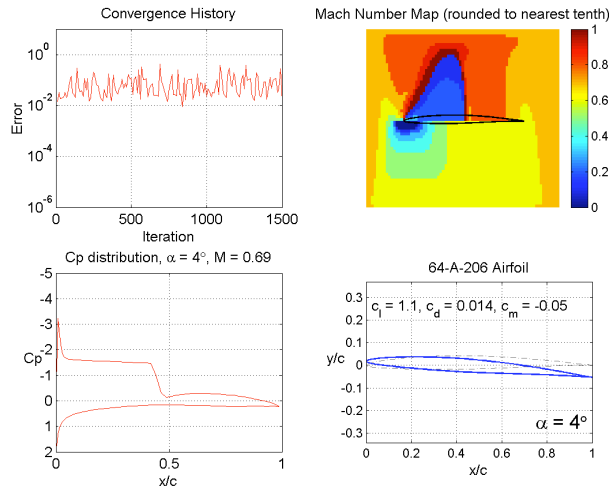
# Beyond the Limits...



Now at 3 degrees you can see the shock appear in the Cp plot and the Mach Map. Notice the data has not converged and the plots have changed from blue to red.



# Beyond the Limits...



And the shock continues to grow at 4 degrees.

# The Overall Design

- LE swept aft to delay drag rise
- TE swept forward to reduce wave drag
- Stealth
  - Edges parallel
  - Canted vertical tail
  - High LE and TE sweep
- Configurations similar in design for cost effectiveness

## Comparison of Variants

- The Navy version has a larger wing/tail
  - Needs more lift because of required lower approach speed
  - $(L/D)_{\max}$  is higher because of higher AR
  - Larger  $C_{L@ (L/D)_{\max}}$  and  $S_{\text{Ref}}$  resulting in a higher cruise altitude
  - Neutral point closer to the cg, resulting in more stability

# Summary of Results

	F-35 A/B	F-35 C
$M_{\text{cruise}}$	0.75	0.75
$C_{L\alpha}$ (rad-1)	3.78	3.55
$C_{M\alpha}$ (rad-1)	0.69	0.35
$x_{np}$ (ft)	27.67	29.1
$x_{cg}$ (ft)	30.53	30.53
SM [%]	-19.3	-9.6
$C_{D0}$	0.0095	0.010
$e$	0.995	0.995
$(L/D)_{\text{max}}$	15.25	15.44
$C_{L@L/D\text{max}}$	0.275	0.310
$h_{\text{cruise}}$ (ft)	19,000	25,600

*Any Questions?*

# References

1. Anderson, J.D., *Introduction to Flight*, 3<sup>rd</sup> edition, McCraw-Hill, 1989
2. Mason, W.H., *Transonic Aerodynamics of Airfoils and Wings*, Notes in Configuration Aerodynamics, Virginia Tech, 4/4/02
3. Mason, W.H., *Drag: An Introduction*, Notes in Applied Computational Aerodynamics, Virginia Tech, January 22 1997
4. <http://www.aerospaceweb.org/aircraft/fighter/f35/>
5. <http://www.aerospaceweb.org/>
6. <http://www.lockheedmartin.com>
7. [http://www.aoe.vt.edu/~mason/Mason\\_f/ConfigAero.html](http://www.aoe.vt.edu/~mason/Mason_f/ConfigAero.html)

# Additional Slides

# Equations

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**Drag divergence Mach number, [2]** 
$$M_{DD} = \frac{\kappa_A}{\cos \Lambda} - \frac{t/c}{\cos^2 \Lambda} - \frac{C_l}{10 \cos^3 \Lambda}$$

where  $\kappa_A$  = airfoil technology factor,  $t/c$  = thickness ratio,  $C_l$  = section lift coefficient,  $\Lambda$  = wing sweep angle.

**Critical Mach number, [2]** 
$$M_{crit} = M_{DD} - \left(\frac{0.1}{80}\right)^{1/3}$$

**Induced drag, [3]** 
$$C_{Di} = \frac{C_L^2}{\pi AR e}$$

where  $C_L$  = lift coefficient,  $AR$  = aspect ratio and  $e$  = Oswald efficiency factor.

# Equations

2 of 2

**Maximum lift to drag ratio**  $\left(\frac{L}{D}\right)_{MAX} = \frac{1}{2} \sqrt{\frac{\pi A R e}{C_{D0}}}$

where L = lift, D = drag and  $C_{D0}$  = skin friction drag.

**Lift coefficient at  $(L/D)_{max}$**   $C_{L,(L/D)_{max}} = \sqrt{\pi A R e C_{D0}}$

**Air pressure at  $(L/D)_{max}$**   $P = \frac{2W}{\gamma M^2 C_{L,(L/D)_{max}} S_R}$

where W = vehicle weight (used W = max TOGW),  $\gamma$  = specific heat ratio of air = 1.4, M = Mach number and  $S_R$  = reference area.

From the air pressure the altitude was found using a table for standard atmosphere in [1].



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- Maximum lift to drag ratio is found when skin friction drag is equal to induced drag.
- Here we are making the assumption that there is no drag due to shocks, i.e. no wave drag. That means that the flow field over the wing is all subsonic.
- The air pressure is found from the definition of lift coefficient and writing the velocity as a function of mach number and the speed of sound. In addition to that assuming a perfect gas and writing the speed of sound as a function of pressure and density.