

## AEROELASTIC DIVERGENCE<sup>1</sup>

Aeroelasticity is the discipline concerned with the mutual interaction of structural, aerodynamic and inertial forces on a flexible solid body moving through a fluid. Aeroelastic problems fall into two major categories:

- **Static problems**, for which time is not a variable (no inertial forces): only aerodynamic-structural interactions. Examples: divergence of wings, control surface effectiveness and reversal, aerodynamic load re-distribution on flexible lifting surfaces.
- **Dynamic problems**, for which time is a variable: aerodynamic structural-inertial interactions. Examples: self-excited vibrations (flutter, interaction with control or propulsion systems), transient response.

In this experiment in the open-throat wind tunnel, we will observe static aeroelasticity and make *subcritical* divergence measurements on a very simple, elastically supported wing model. “subcritical” implies that a divergence speed is determined without the necessity of actually subjecting the model to that airspeed and possibly breaking it.

We will examine the divergence of a symmetrical, two-dimensional airfoil restrained by a rotation spring as sketched in Figure 1. Aerodynamic force causes an elastic angle of attack corresponding to structural twisting of a real wing. To find the relationship between  $\alpha$  and airspeed  $V$  (or dynamic pressure  $q = \frac{1}{2}\rho V^2$ ), we sum the moments about the hinge. The angle of attack is assumed to be sufficiently small that  $\cos \alpha \cong 1$  and  $\sin \alpha \cong \alpha$ .

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<sup>1</sup>Lab description courtesy of Dr. W.L. Hallauer.

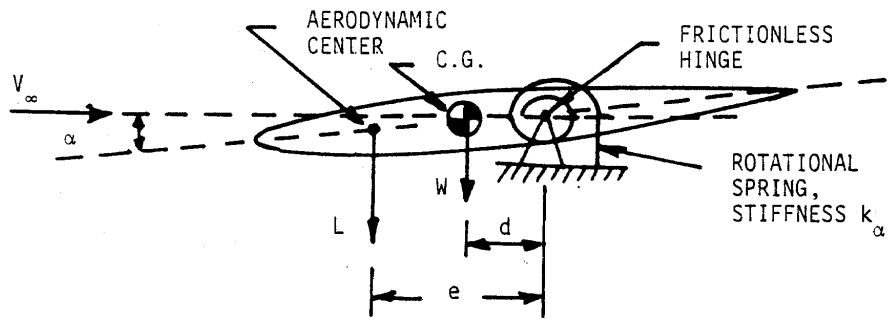


Figure 1: MODEL OF A WING

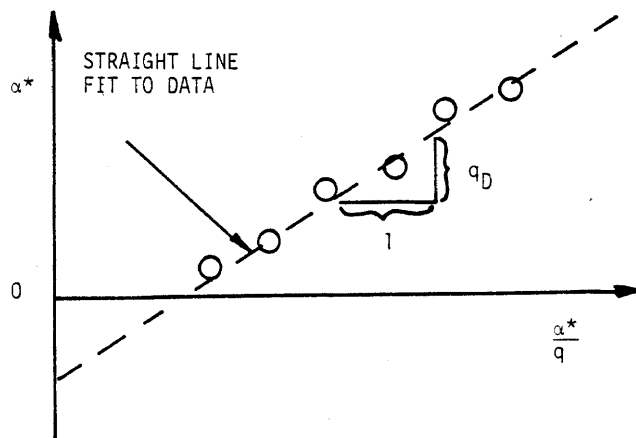


Figure 2: DIVERGENCE SOUTHWELL PLOT

$$+ \Sigma M = 0 : Le + Wd - k_\alpha \alpha = 0 \quad (1)$$

where  $L = qSC_\ell = qSM_0\alpha$ ,

$S$  = planform area,

$C_\ell$  = lift coefficient

$M_0$  = lift curve slope.

Solving for  $\alpha$ ,

$$(qSM_0e - k_\alpha) \alpha + Wd = 0, \quad (2)$$

or

$$\alpha = \frac{Wd}{k_\alpha - qSM_0e} \quad (3)$$

Now define

$\alpha_z = Wd/k_\alpha$ , zero airspeed angle of attack due only to airfoil weight,

$q_D = \frac{k_\alpha}{eSM_0}$ , divergence dynamic pressure.

So

$$\alpha = \frac{\alpha_z}{1 - \frac{q}{q_D}} \quad (4)$$

A plot of  $\alpha$  versus  $q$  is hyperbolic with angle of attack becoming infinite at  $q_D$ . This is, of course, physically impossible, and the airfoil will stall or the spring will break when twist becomes large. Because of the latter possibility (i.e., structural failure), the value of  $q_D$  is an important quantity relative to the structural integrity of a wing. We would like to be able to predict  $q_D$  in a non-destructive experimental manner. Since it is difficult to obtain experimental data points which will accurately fit a hyperbola and use them to predict the asymptote it is necessary to transform equation 4 into a straight line form. Define  $\alpha^* = \alpha - \alpha_z$ , the angle measured relative to the zero airspeed angle. Then

$$\alpha = \alpha^* + \alpha_z = \frac{\alpha_z}{1 - \frac{q}{q_D}} \quad (5)$$

or

$$\alpha^* = \frac{\alpha_z - \alpha_z(1 - \frac{q}{q_D})}{1 - \frac{q}{q_D}} = \frac{\alpha_z \frac{q}{q_D}}{1 - \frac{q}{q_D}}$$

or

$$\alpha^* - \alpha^* \left( \frac{q}{q_D} \right) = \alpha_z \frac{q}{q_D}$$

or

$$q_D \left( \frac{\alpha^*}{q} \right) - \alpha^* = \alpha_z$$

or, finally,

$$\alpha^* = q_D \left( \frac{\alpha^*}{q} \right) - \alpha_z \quad (6)$$

Equation 6 has the form of a straight line ( $y = mx + b$ ) where  $y = \alpha^*$ ,  $m = q_D$ ,  $x = \frac{\alpha^*}{q}$  and  $b = -z$ . This allows us to use a non-destructive testing technique to find points along a straight line and the resulting curve slope will give us the divergence  $q(q_D)$ . The resulting plot is known as a Divergence Southwell Plot such as that shown in Figure 2. Hence, if the airfoil-spring combination used in the lab behaves according to linear theory,  $q_D$  can be measured as the slope of the straight line. The experimental setup used in the three-foot tunnel is shown in Figure 3. The following experimental procedure is based on Figure 3 (the value for  $e$  will be given by your lab instructor):

- A. Stiffness measurement. Tunnel off. Set  $L$  as assigned. Set  $\alpha_z = 2 - 4^\circ$  (nose down) and note the pointer reading. Hang weights  $W = 0, 1, 2, 3$  lbs. Calculate and carefully plot  $M_0$  versus  $\alpha^*$ . Is the final unloaded  $\alpha_z$  (pointer reading nearly the same as the initial  $\alpha_z$ ?) If not, a fastener may have slipped during loading, and you should tighten all nuts and repeat the experiment (The same applies for the divergence runs below.) Fit a straight line through the data points (by eye, or by least-squares if you know how,) and calculate an experimental  $k_\alpha$ . Compare with the corresponding theoretical value.
- B. First divergence measurement.  $L$  as in part A. Tunnel on, but always be prepared to turn it off quickly in case a fastener slips or something breaks - it has happened before. Set  $\alpha_z = 2 - 4^\circ$  (nose down) and note the pointer reading. Set the tunnel manometer head  $h$  carefully, and measure pointer reading  $d$  versus  $h$ ; start at  $h = 0.6''$  and increase in  $0.2''$  increments, but don't exceed  $2''$  and don't produce angles such that the pointer goes off scale. Calculate  $\alpha^*$  in degrees,  $q(psf) = 5.2h''$ ,  $\frac{\alpha^*}{q}$ , and plot the data on a divergence Southwell graph. Fit a straight line on the graph, calculate the experiential  $q_D$ , and compare with the corresponding theoretical value before proceeding to part C.
- C. Second divergence measurement. Change  $L$  as assigned, and repeat the procedure of part B.

1. In your lab report, discuss the following questions:
2. Do the data fit along straight lines as predicted by theory?
3. Does  $q_D$  vary with  $k_\alpha$  as predicted by theory?
4. What are the major sources of discrepancy and error, if any?

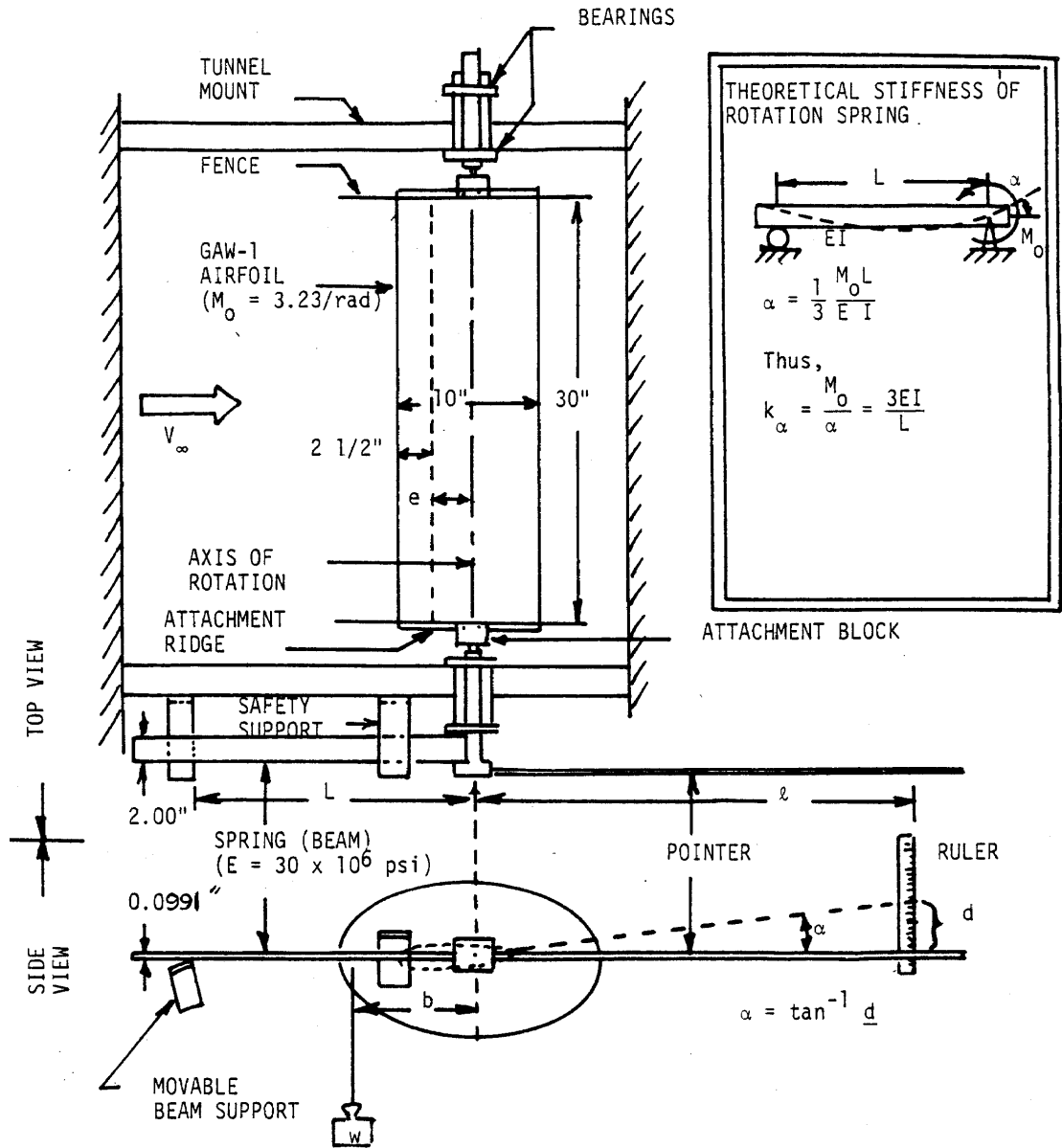


Figure 3: DIVERGENCE TEST APPARATUS