NOTES FOR DESIGN OF EXPERIMENTS

Required References:

Course Manual for AOE 3054, available at http://www.aoe.vt.edu/aoe3054/

Chapter 2 and Class Handouts for Classes 6 and 7. It is assumed that each student is familar with this material since it was presented in AOE 3054.

Optional References:

Doebelin, E. O. 1995 **Engineering Experimentation**: Planning, Execution, and Reporting, McGraw-Hill Book Co., NY.

Holman, J. P. 2000 "Design of Experiments," chapter 16, pp.638 - 679, **Experimental Methods for Engineers**. McGraw-Hill Book Co., NY.

Wheeler, A. J. and Ganji, A. R. 1996 "Guidelines for Planning and Documenting Experiments," chapter 12, pp. 360 - 384, **Introduction to Engineering Experimentation**, Prentice Hall, Englewood Cliffs, N J.

OVERVIEW OF DESIGN OF AN EXPERIMENT

Systematic approach with following phases:

1. **Problem definition**

2. **Experiment design**

3. **Experiment construction and development**

4. **Data gathering**

5. **Analysis of data**

6. **Interpretation of results and reporting**

1. PROBLEM DEFINITION

Assumption: Technical need established; non-experimental approaches inadequate.

Are all relevant physical phenomena known?

If so, a more limited amount and type of data may be required; *this may reduce the time and cost.* <u>Run risk of missing important information</u>.

Example: testing of flowmeter. Rotating vanes of turbine wheel generate electrical pulses through magnetic pickup; frequency is proportional to flow through passage; frequency to voltage converter (See figure).

If not, then a more detailed amount of data may be required to understand the roles of the different phenomena; *this will increase time and cost but will define the problem in more detail.* <u>Safer approach to not miss effects</u>.

Example: flowmeter failed with fluttering vanes that caused fatigue of vanes; test did not have enough instrumentation to detect the flutter.

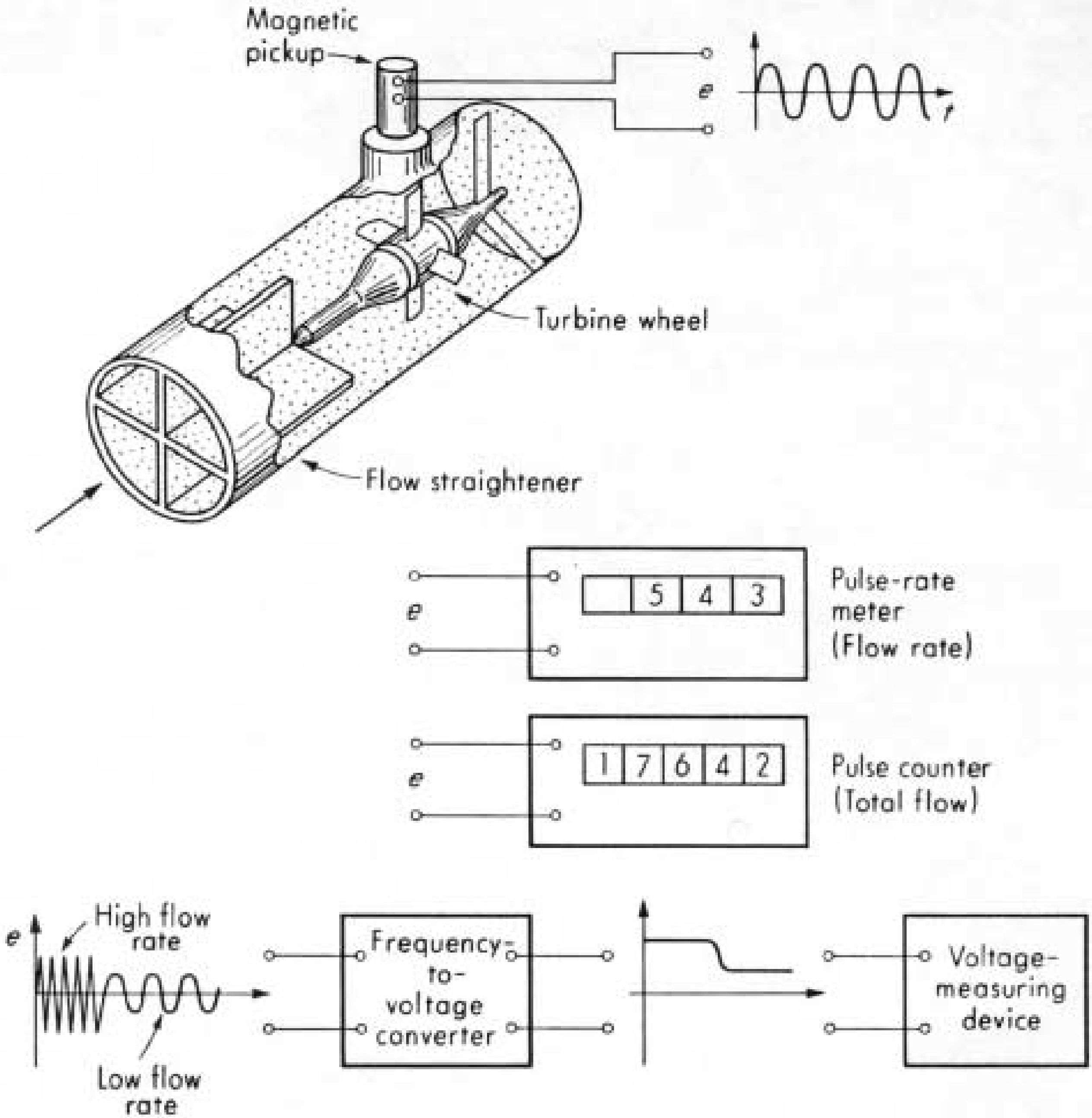


Figure 7.35 Turbine flowmeter.

2. EXPERIMENT DESIGN

➜ Preliminary Design - scoping type of study with costs, resulting in a written Proposal to financial sponsor

➜ Final Design - refined design with all detailed issues resolved

May include the following major components, <u>which are interactive</u>:

A. <u>Search for information</u>

- **Literature survey for ideas from similar previous experiments**

B. <u>Determination of experimental approach</u>

- **Have several approaches - let other factors guide selection of final approach; use brainstorming sessions to get ideas**
- **May need to design cheaper "pilot" experiments to prove concepts for approach selected**

C. <u>Specification of measured variables</u>

- **Make sure that the data respond to the project objectives ● Define the measurements to be made and their locations**

D. <u>Determination of the analytical model(s) used to analyze the data</u>

Develop computer programs for analysis of data

E. <u>Estimation of experimental uncertainties</u>

- **Use data reduction computer programs to estimate uncertainties - perturbation or "jitter" analysis**

(See *Glossary of Other Terms* for more detail.)

- **High uncertainties may require change of approach**
- **Use uncertainty analysis to specify the uncertainty for each instrument - use the "method of equal effects", which is that each instrument contributes the same amount to the final uncertainty. (See *Glossary of Other Terms*)**
- **Real uncertainties are NEVER less than design uncertainties - square of uncertainties are always positive and additive!**

F. <u>Considerations for the Selection of instruments</u>

- **Use of available equipment and personnel experienced in their use.**
- **Cost of new instruments and sensors for acceptable uncertainty and sensitivity with lack of sensitivity to other independent variables.**
- **Accuracy and precision are limited by the <u>hysteresis</u> (See *Glossary of Other Terms*) of a transducer or instrument**
- **Costs of calibration (equipment, personnel, time)**
- **Instrument and sensor maintenance costs**
- **Use more than one instrument or sensor to measure same quantity for redundant independent measurement that reduces uncertainties**

G. <u>Determination of the test matrix and sequence</u>

- **Remember - test time is $$$.**
- **Use dimensional analysis to reduce number of test points and to determine the apparatus scale.**
- **Range of values of independent variables (factors) to be tested.**

- **Use understanding of physical phenomena to help select number of test values.**
- **Some tests may need to be done first to define later tests.**
- **Use a number of <u>replications</u> of each test to reduce uncertainties using <u>statistical techniques.</u> (<u>Replication</u> is a repeated test at a different later time with different instruments and/or different operators. <u>Repetition</u> is conduct of the same test immediately with the same equipment, setup, and personnel.)**
- **Use random order of test sequence over range of variable when possible in order to reduce artificial trends in data, e.g., no correlation of data with time of day! Replicate or repeat in a different random order.**

H. <u>Determination of time schedule and costs</u>

- **Define a set of tasks and deadlines**
- **Estimate the amount of work in man hours, the required skills, and the time for each task**
- **Develop scheduling network, using an "activities network" or Gantt chart to define the "critical path" for project completion. (See attached figure and** http://www.aoe.vt.edu/aoe/faculty/Mason_f/SD1VGs.html **lecture No. 4, charts 19-26, for a few Gantt charts.)**
- **Develop budget of all costs for the complete activity (design, construction, analysis, and reporting): equipment, supplies, salaries, benefits (≈30% of salaries), and overhead for doing business (≈100% of salaries); should be realistic and frugal to win contract, but provide some additional funds for unforseen costs**

I. <u>Mechanical design of the test apparatus</u>

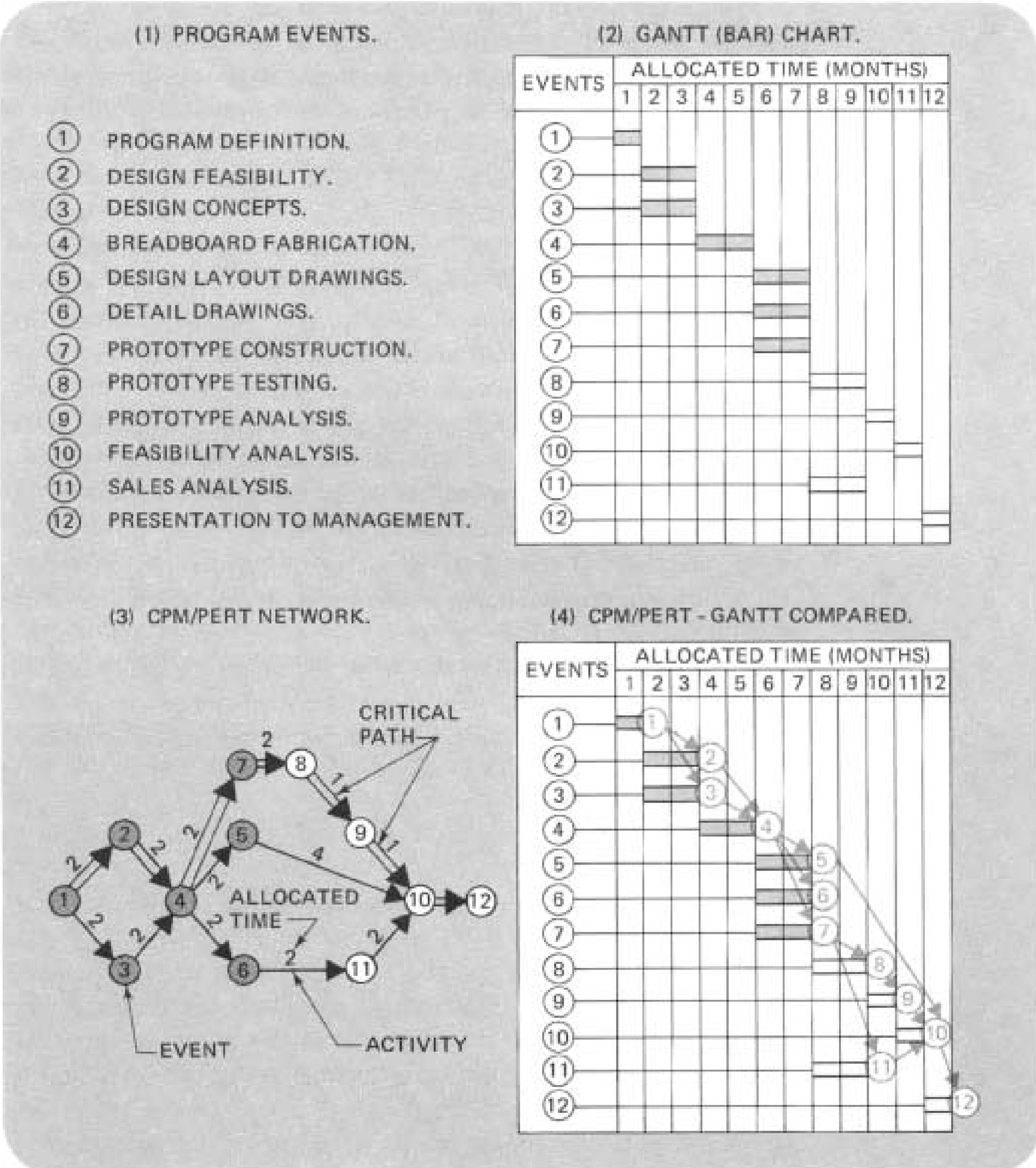


Figure 7-1 **PROGRAM CONTROL TECHNIQUES**

- **Use dimensional analysis to determine the apparatus scale**
- **Drawings required of all components to be constructed**

J. <u>Specification of test procedure</u>
Human and equipment safety issues

Example: VASCIC Building Wind Tunnel Tests (Described at the end of this Section)

3. EXPERIMENT CONSTRUCTION AND DEVELOPMENT

A. Procure required commercial off-the-shelf (COTS) equipment

B. Have workshop construct custom-designed components (usually more expensive to build than buy components)

C. Calibrate individual sensors and instruments under known conditions to eliminate repeatable systematic errors. Use calibration instruments that have <u>an order of magnitude lower uncertainty</u> than instruments that are being calibrated.

D. Assemble test apparatus and conduct preliminary or "shakedown" tests

- **Use ALL data reduction computer programs.**
- **Make sure all instruments read correct values under known conditions.**
- **Use any applicable physical principles to perform overall check on validity of experiment.**
- **Discover problems (e.g., flow leaks, faulty electrical connections, data reduction program errors, etc.).**

- **Perform uncertainty analysis of preliminary data to confirm sources of uncertainties and discover additional uncertainties (which are actually present!).**

E. Make modifications to equipment or even experimental approach; modify test matrix and sequence; modify data reduction programs

4 &5. DATA GATHERING AND ANALYSIS

- **Gather data according to revised test matrix and sequence**
- **Perform computer data reduction, including uncertainty analysis BEFORE ALL DATA HAVE BEEN TAKEN.**
- **If necessary, modify procedures and retake data for conditions when questionable data were acquired.**

6. INTERPRETATION OF RESULTS AND REPORTING

- **Make sure that the data and results respond to the project objectives**
- **Develop logical reasons to explain data trends; explain anomalous data**
- **Compare and validate results with results from previous or similar experiments; make sure results are "reasonable"**
- **Prepare complete report that includes:**

  A. Concise summary that answers the major questions for which the experiments were done

  B. Complete description of all facilities and equipment used; present calibration data and uncertainties for all measurements

C. Detailed discussion of the results
D. Complete set of conclusions that provide answers to all questions for which the experiments were done.

EXAMPLE

A WIND TUNNEL STUDY OF THE ATMOSPHERIC WIND FLOW OVER A 1/100 SCALE MODEL OF THE VIRGINIA ADVANCED SHIPBUILDING AND CARRIER INTEGRATION CENTER (VASCIC)

Roger L. Simpson, Professor

Research Engineers and Task Leaders

John Fussell - Forces and Moments, Flow Visualization

Jacob George - Mean Surface Pressure Distributions

Michael Goody - Surface Pressure Fluctuations

Yu Wang - Hot-wire Anemometer Wind Tunnel Surveys

(all experienced in their tasks)

Assisted by

Troy Jones

Serhat Hosder

Hao Long

Rulong Ma

Jana Schwartz

Model Construction

J. Greg Dudding, Bruce Stanger, Steve McClellan

William Mish, William Moon

Warren Shelor - Owens Dining Hall large oven

Greg Bandy - Stability Wind Tunnel Technician

Department of Aerospace and Ocean Engineering

Virginia Polytechnic Institute and State University

Blacksburg, VA 24061- 0203

Funded by

Clark and Nexsen, Engineers and Architects

Norfolk, Virginia

Figure 1.1 Clark and Nexsen architectural drawings of views of VASCIC toward the west from land (top) and toward the east from the James River (bottom).

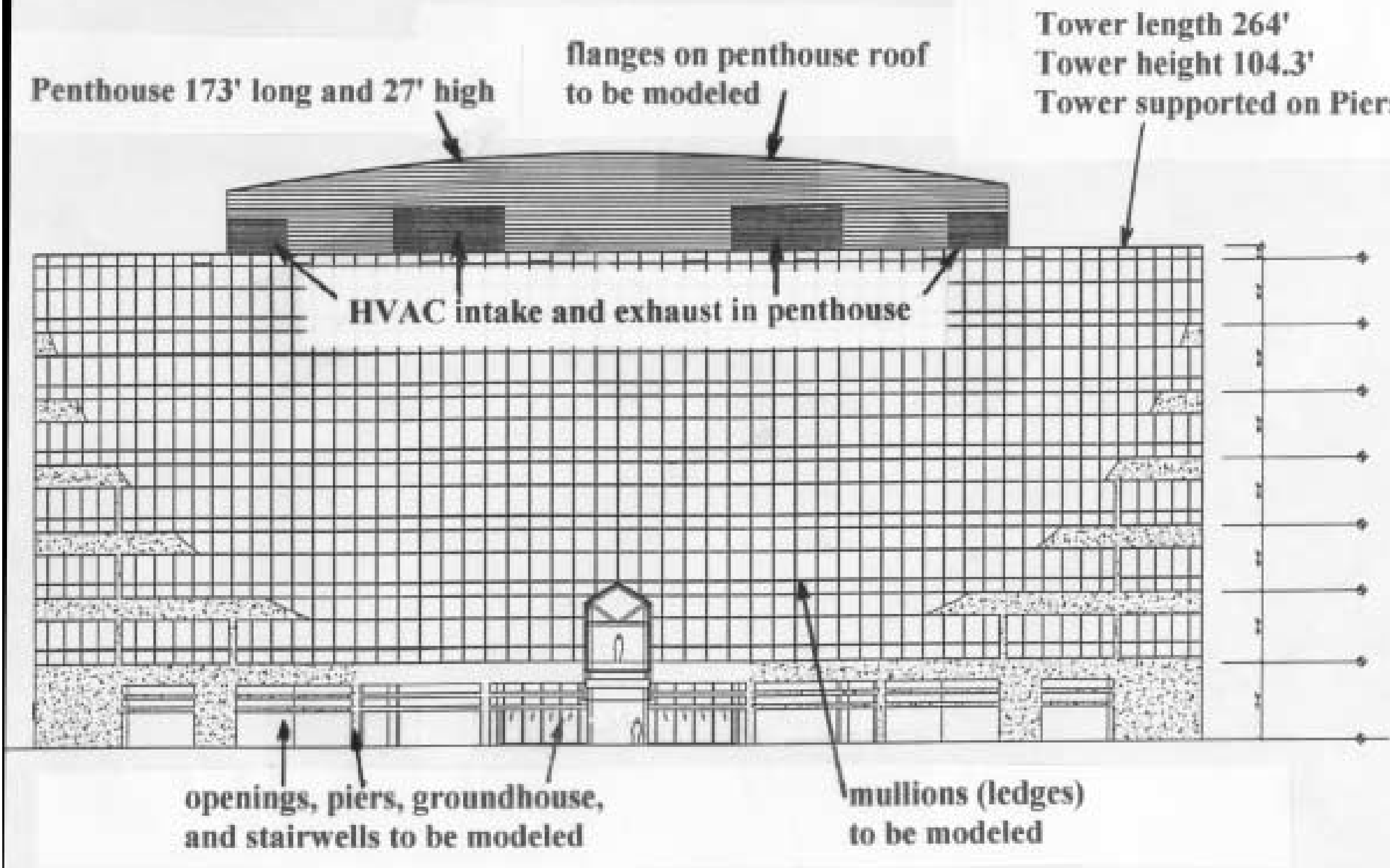


Figure 1.3 Clark and Nexsen architectural drawing from the south of the VASCIC tower, showing the ground level open area and the penthouse on top of the tower.

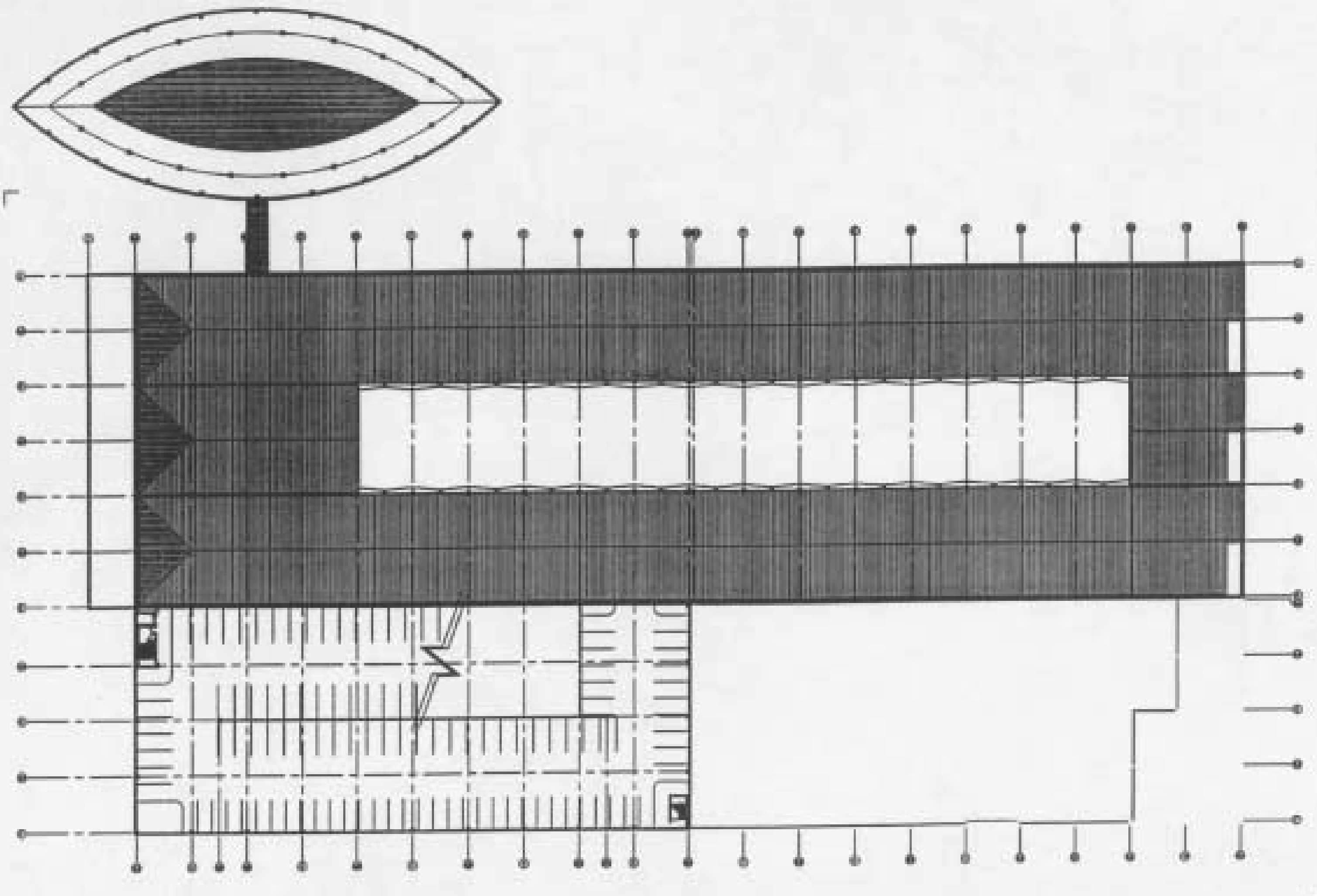


Figure 1.4 Clark and Nexsen roof plan view architectural drawing of the VASCIC building complex showing the tower (top), laboratory (middle), and garage (bottom) buildings.

THE PROBLEM - Designing a Building for Hurricanes

- **Because of the unusual shape of this tower, quantitative information on the total wind loadings on the tower and the local peak pressures and frequencies on the glass panels are needed for various wind directions.**

- **Flow patterns around the tower strongly influence the locations of the peak pressures and fluctuations. Identify locations of separations, vortical flow structures, unsteady flow regions.**

- **Unfavorable wind flow patterns may be produced for some wind directions. Such unfavorable patterns could influence the location of decorative gardens, fountains, and pedestrian walkways and affect HVAC on roof.**

APPROACH TO OBTAIN INFORMATION

Measurements on a 1/100 Scale Model in the Virginia Tech Stability Wind Tunnel

- **Proven simulated atmospheric boundary layer methods to produce results applicable to full scale; spires and blocks to create semi-logarithmic mean velocity profile and small-scale turbulence; slotted-wall test section reduces side-wall interference or wind tunnel blockage effects; computational fluid dynamics produces too uncertain results.**

- **Flow separations on model are weakly influenced by Reynolds number.**

- **Simple design - intersection of cylinders, inexpensive, easy to construct, accommodates instrumentation, no artificial flow interference; laboratory buildings from wood; garage machined from plexiglas; model rotatable on simple turntable in wind tunnel to**

vary wind direction.

- **Use simplest available instrumentation**

<u>RESOURCES</u>

- **Available instrumentation and data reduction codes:**
  - ➜ **scanivalve system for mean pressures over model surfaces; over 300 static pressure taps on 1/4 of symmetric model.**
  - ➜ **calibrated transducers for surface pressure fluctuations in separated flow regions.**
  - ➜ **calibrated unsteady force loadcells (See attached figure).**
  - ➜ **flow visualization:**
    - □ **oil-flows on horizontal surfaces.**
    - □ **yarn tufts on thread mounted on surface to show separations on vertical surfaces; tufts oscillate violently in separated flow region**
    - □ **neutrally buoyant helium-filled soap bubbles that follow flow.**

- **Experienced personnel to construct model.**

- **Personnel experienced in the use of a given type of instrumentation and in the interpretation and reporting of results.**

<u>CONSTRAINTS</u>

- **COST - fixed price contract.**

- **TIME - 6 months, including model design, construction, tests and report.**

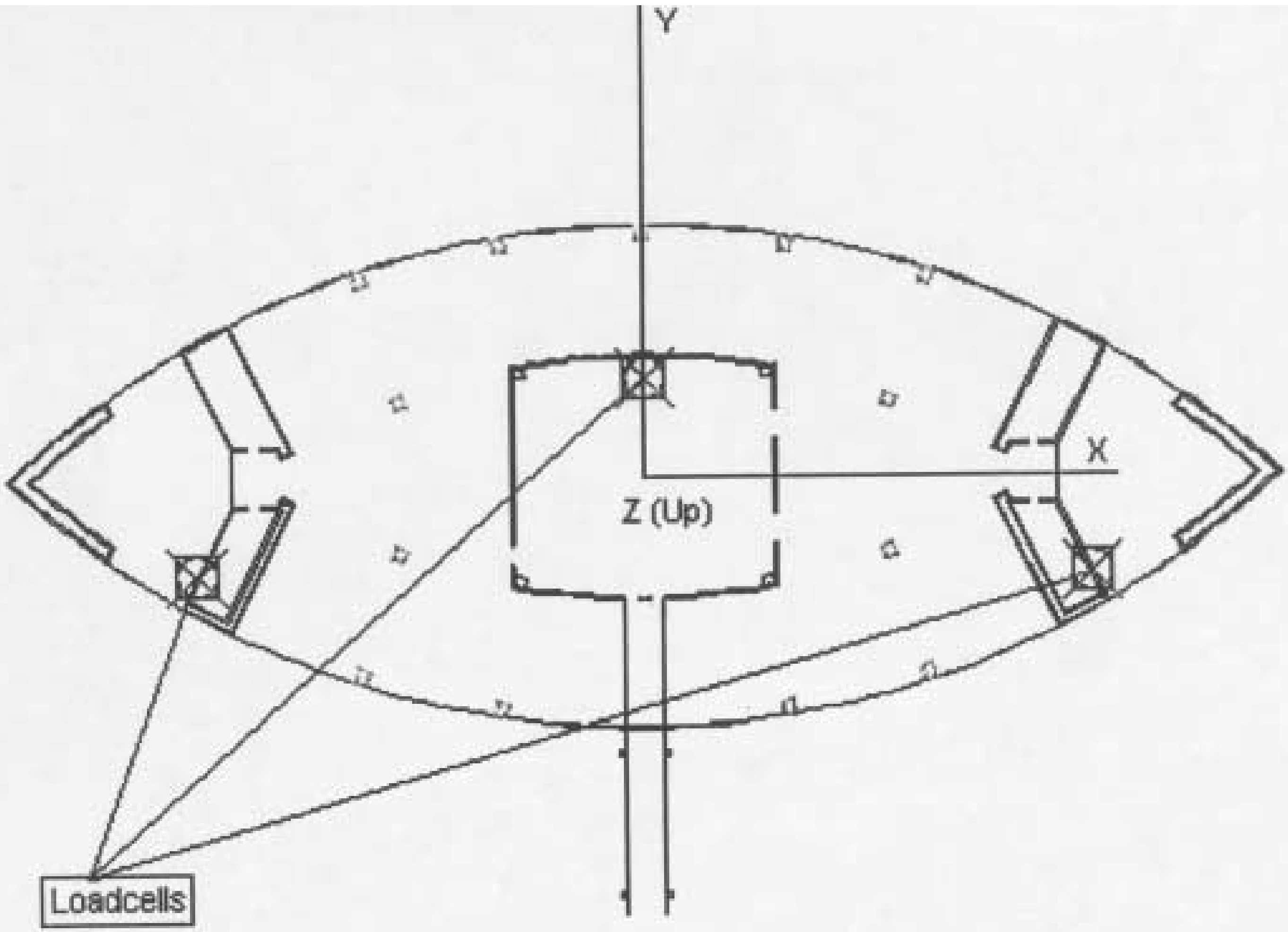


Figure 5.2: Loadcell locations for the force and moment tests.

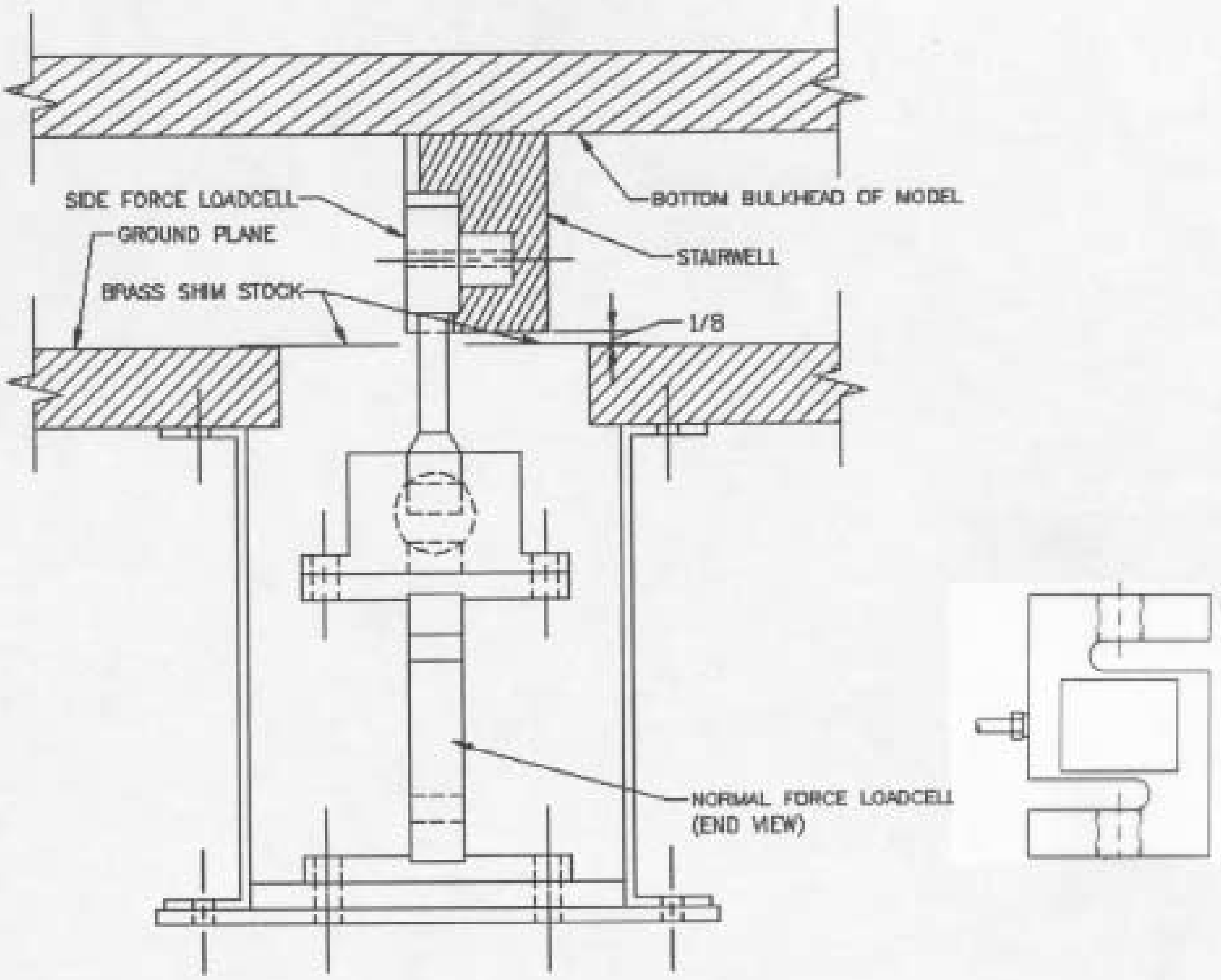


Figure 5.3: A diagram showing how loadcells were mounted for the force and moment tests.

CONCLUSIONS FROM TESTS

- **Forces (see attached figure) , moments, peak and mean pressures, pressure frequency spectra, unsteady force spectra, and flow visualization data were obtained for 12 different wind directions over the model VASCIC building complex.**

- **No strong Reynolds number effects.**

- **No narrow frequency band periodic flow phenomena from Tower.**

- **Most energetic pressure fluctuation frequencies: $0.1 < nL/U_{ref} < 0.5$. (N = frequency; L = length of building; U_{ref} = approach velocity)**

- **Windows and exterior panels will be subjected to fatigue by pressure fluctuations at resonant frequencies of building materials.**

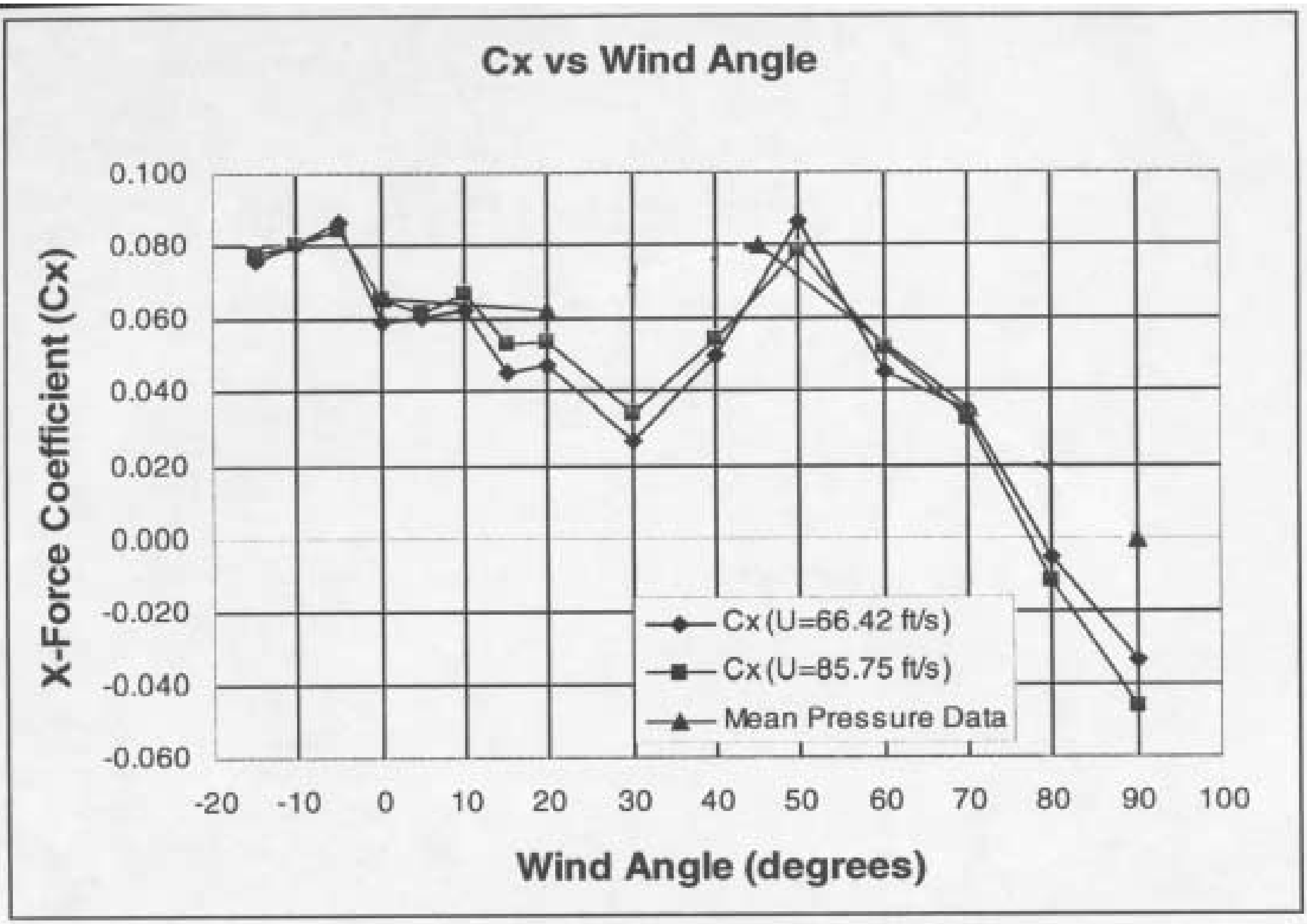


Figure 5.4 Non-dimensional X force verses wind angle for the second set of tests.

Glossary of Terms Related to Measurement Accuracy and Uncertainty
from U.S. National Institute of Standards and Technology NIST – TN/297, 1994

Accuracy. The closeness of the agreement between the result of a measurement and the value of the specific quantity subject to measurement, i.e., the measurand. Although most equipment manufacturers still use the term as a tolerance in their specifications, NIST and other international standards bodies have classified it as a qualitative concept not to be used quantitatively. The current uniform approach is to report a measurement result accompanied by a quantitative statement of its uncertainty.

Error. The result of a measurement minus the value of the measurand.

Measurand. The true value of the specific quantity subject to measurement.

Precision. The closeness of agreement between independent test results obtained under stipulated conditions. Precision is a qualitative term used in the context of repeatability or reproducibility and should never be used interchangeably with accuracy.

Repeatability. The closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement.

Reproducibility. The closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement.

Resolution. A measure of the smallest portion of the signal that can be observed. For example, a thermometer with a display that reads to three decimal places would have a resolution of 0.001°C. In general, the resolution of an instrument has a better rating than its accuracy.

Sensitivity. The smallest detectable change in a measurement. The ultimate sensitivity of a measuring instrument depends both on its resolution and the lowest measurement range. Also ∂(output variable)/∂(input variable).

Uncertainty. The estimated possible deviation of the result of measurement from its actual value within some given odds, usually 20:1. The uncertainty of the result of a measurement generally consists of several components that may be grouped into two categories according to the method used to estimate their numerical values: **A**. those evaluated by statistical methods; **B**. those evaluated by other means. Uncertainty and error are not to be used interchangeably.

Glossary of Other Terms

Bias or Systematic Error - repeatable fixed errors which can be removed by calibration.

Random error – errors which cannot be removed by calibration.

Calibration – The intercomparison between 2 instruments/sensors, one of which is a certified standard of known accuracy, in order to correct the accuracy of the item being calibrated. Calibration can eliminate the bias or systematic errors.

Hysteresis – Path dependent output for the same input; the maximum output and minimum output for a given input occur within the hysteresis loop. Accuracy and precision are limited by the hysteresis of the transducer. See attached figure for example of hysteresis.

Threshold – minimum detectable value of input quantity.

Gaussian Normal Curve of Error (p. 71, Holman)– $P(x) = \exp[-(x - x_m)^2/2\,\sigma^2]/\sigma(2\pi)$, where x_m is the mean value and σ is the standard deviation. The assumption is that there are many small errors that may be equally positive or negative that contribute to the final error. 68.3% of values fall between +/- 1σ of x_m; 95% of values fall between +/- 1.96σ of x_m (20:1 odds); 99.7% of values fall within +/- 3σ of x_m.

Non-gaussian Probability Distribution – not all statistical processes follow the gaussian normal curve of error; other models have been found to be common for particular types of phenomena.

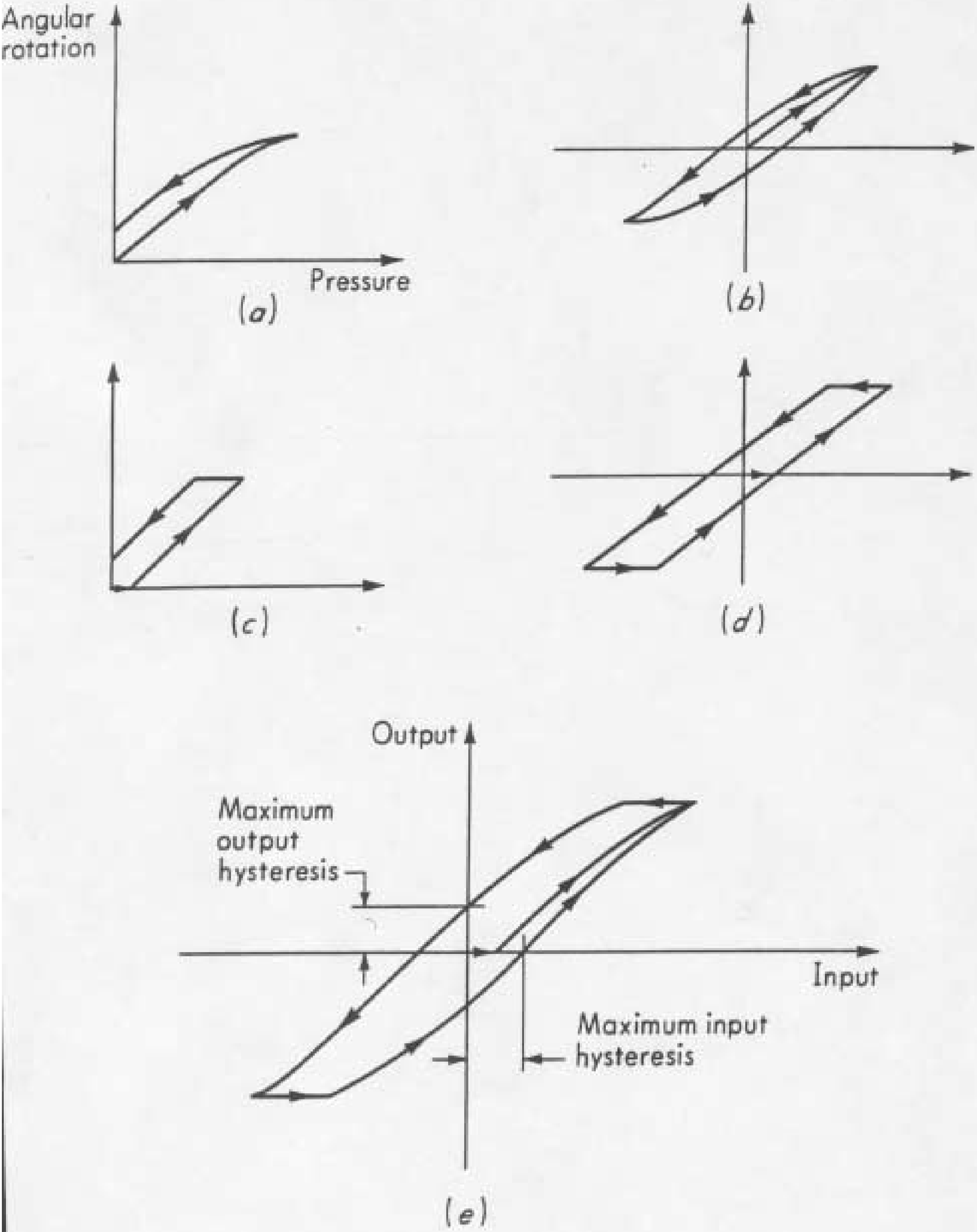
Chi-squared Goodness of Fit (p. 84, Holman) – mean square deviation χ^2 of actual probability distribution of experimental data from an assumed distribution; acceptable χ^2 values are determined by the number of degrees of freedom, which is the number of bins in the experimental probability distribution minus the number of constraints.

Chauvenet's Criterion for Rejection of Outlying Points (p. 78, Holman) – For n readings, a reading may be rejected if the probability of obtaining that deviation is less than 1/2n. For example, 10 readings permit rejection of deviations greater than 1.96σ since 1/20 of the readings are permitted to have a greater deviation; 166 readings permit rejection of deviations greater than 3σ since 1/333 of the readings are permitted to have a greater deviation.

Least-squares Fit (p. 91, Holman and in most calculus books) – a fit of a selected model equation (often a straight line) to experimental data with a minimum sum of squares of the deviations.

Correlation coefficient R between data and curve fit (p. 95, Holman) – Given data values of y for given values of x, then the correlation coefficient $R = [\,1 - \sigma^2_{xy}/\sigma^2_{y}]^{1/2}$, where σ_{xy} is the standard deviation between the data and the curve fit and σ_y is the standard deviation of the data and the mean value of the data.

"Jitter" analysis; Evaluation of Uncertainties for Complicated Data Reduction (p.60, Holman) - If $S = S(x_1, x_2, x_3, \ldots x_n)$ is calculated by the computer program with input of quantities x_1, x_2, x_3,



[redacted] Hysteresis effects. (Magnitude exaggerated for graphical clarity.)

$_{\ldots}$ $x_{n,,}$ then the sensitivities $\partial S/\partial x_n \approx [S(x_n+\Delta x_n) - S(x_n)]/\Delta x_n$, etc. can be calculated by the program for small Δx_n and

$\delta S = [\sum\{(\partial S/\partial x_n)\delta x_n\}^2]^{1/2}$, where δS , δx_1, δx_2, …, δx_n are the uncertainties of the calculated quantity S and the input quantities x_1 , x_2, x_3, ... x_n, each at the same odds.

Method of Equal Effects for Design If $S = S(x_1, x_2, x_3, \ldots x_n)$ with input of quantities x_1 , x_2, x_3, ... x_n, then the uncertainty is $\boldsymbol{\delta S} = [\sum\{(\partial S/\partial x_n)\delta x_n\}^2]^{1/2}$, where δS , δx_1, δx_2, **…,** δx_n are the uncertainties of the calculated quantity S and the input quantities x_1 , x_2, x_3, ... x_n, each at the same odds. The variables that contribute the most to the uncertainty of S dominate the uncertainty; the variables that contribute little to the uncertainty in S could have even greater uncertainty and still not significantly increase the uncertainty in S. (**See example on next page**.)

The method of equal effects for the design of a measurement system recognizes that each input variable should have the same contribution to the uncertainty of the output variable. In other words, $\{(\partial S/\partial x_n)\delta x_n\}^2 = \{(\partial S/\partial x_m)\delta x_m\}^2$ for each input variable n not equal to m. If one designs an experiment for an uncertainty δS and has M input variables, then each input variable should contribute $\delta S/(M)^{1/2}$ to the uncertainty.

Uncertainty of a mean (p.98, Holman) – given n > 10 measurements of the same quantity in a set, each with uncertainty δx for a given odds, then the uncertainty of the mean $\delta x_m = \delta x/\sqrt{n}$.

Student's t distribution – (p.99, Holman) - ("Student" was the pen name of Wm. Gosset) - for a small number of samples n, the uncertainty of the mean is $\delta x_m = (t)\sigma/\sqrt{n}$, where "t" is a number from a distribution function depending on the odds and the number of samples. For very large n, t approaches values for a gaussian distribution. For example, for $n \rightarrow \infty$, at 20:1 odds, the t = 1.96 as for the gaussian distribution and the result is the same as for the **Uncertainty of a mean** above.

As an example, suppose we are manufacturing capillary tube flow resistances for use in pneumatic control systems. We purchase capillary tubing from a supplier whose quality control department tells us that the tube inside diameter has a mean value $\bar{d}$ of 0.0100 in and standard deviation σ_d of 0.0005 in. We cut the tube to length in our own cutoff machines, which produce a length with mean value $\bar{l} = 10.00$ in and $\sigma_l = 0.05$ in. To meet specifications, our pneumatic controller can tolerate a flow-resistance standard deviation of no more than 10 percent of the mean value. Will we meet this specification with the values quoted above? The formula for flow resistance is

$$R_{fl} = \frac{p}{q} = \frac{\text{psi}}{\text{in}^3/\text{s}} = \frac{128\,\mu l}{\pi d^4} \tag{2.62}$$

$$\frac{\partial R_{fl}}{\partial l} = \frac{128\mu}{\pi d^4} = \frac{128\mu}{\pi} 10^8 \qquad \frac{\partial R_{fl}}{\partial d} = \frac{(-4)(128\mu l)}{\pi d^5} = \frac{-128\mu}{\pi} \times 40 \times 10^{10}$$

$$\sigma_{R_{fl}}^2 = \left(\frac{128\mu}{\pi}\right)^2 [(10^{16} \times 25 \times 10^{-4}) + (1600 \times 10^{20} \times 25 \times 10^{-8})] \tag{2.63}$$

$$\sigma_{R_{fl}} = \sqrt{\left(\frac{128\mu}{\pi}\right)^2 (\underbrace{25 \times 10^{12}}_{\text{effect of } l} + \underbrace{1600 \times 25 \times 10^{12}}_{\text{effect of } d})} = \frac{128\mu}{\pi}(200 \times 10^6) \tag{2.64}$$

Since we want $\sigma_{R_{fl}}$ as a percentage of R_{fl},

$$\frac{\sigma_{R_{fl}}}{R_{fl}} = \frac{(128\mu/\pi)(200 \times 10^6)}{(128\mu/\pi)(10/10^{-8})} = 0.20 = 20\% \tag{2.65}$$

Note that a σ_d of 5 percent of the mean value of (0.0005 out of 0.01) and a σ_l of 0.5 percent (0.05 out of 10.0) cause a $\sigma_{R_{fl}}$ of 20 percent! The analysis clearly shows that the tube diameter, not length, is the main source of variability because of the d^4 term in R_{fl}. Note from Eq. (2.64) that while σ_d (as a percentage of d) is 10 times σ_l, the *effect* of σ_d on $\sigma_{R_{fl}}$ is 40 times the effect of σ_l, due to the d^4 term.